Validation of a coupled fluid/structure solver and its application to novel flutter solutions

By

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A coupled fluid-structure interaction solver capability is developed and validated. A high fidelity fluids solver, Loci-Chem, is coupled with a finite-element structural dynamics toolkit, MAST. The coupled solver is validated for the prediction of several panel instability cases in uniform flows and in the presence of an impinging shock for a range of subsonic and supersonic Mach numbers, dynamic pressures, and pressure ratios. The panel deflections and limit-cycle oscillation amplitudes, frequencies, and bifurcation point predictions compare very well with benchmark results for 2D simulations. The same procedures outlined in the validation study have been applied to simulations of varying dynamic pressures at $M = 2$ for an impinging oblique shockwave. The influence of inviscid, laminar and turbulent boundary layer profiles on the development of flow field characteristics has been analyzed, and laminar predictions characterized by a large flow separation results in vastly different behavior than that of traditional flutter.
Key words: flutter, shock impingement, shock-boundary layer interaction, fluid-structure interactions, aeroelasticity, bifurcation analysis
DEDICATION

To all of my friends and family.
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The findings and opinions in this thesis belong solely to the author, and are not necessarily those of the sponsor.
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A  amplitude

\( a \)  fluid speed of sound, \( \frac{m}{s} \)

\( C_f \)  friction coefficient

\( (\cdot)_c \)  chord location

\( \tilde{(\cdot)} \)  filtered quantity

\( \hat{(\cdot)} \)  Farve averaged quantity[12]

\( D \)  plate rigidity, \( J \)

\( E_s \)  Young’s modulus, \( Pa \)

\( f \)  flutter frequency, \( Hz \)

\( H \)  plate thickness, \( m \)

\( K_f \)  nondimensional frequency

\( k_{cond} \)  laminar thermal conductivity, \( \frac{W}{m\cdot K} \)

\( L \)  plate length, \( m \)

\( M \)  Mach number

\( P \)  fluid pressure, \( Pa \)

\( Pr \)  Prandtl number

\( Re \)  Reynolds number, \( \frac{\rho u L}{\mu} \)

\( R_s \)  specific gas constant, \( \frac{m^2}{s^2 K} \)

\( St \)  Strouhal number, \( \frac{f L}{U} \)

\( s \)  nodal spacing, \( m \)

\( T \)  fluid temperature, \( K \)

\( u, u_i \)  flow speed in the \( i^{th} \) coordinate direction, \( \frac{m}{s} \)
$U_f$ friction velocity, $\frac{m}{s}$

$W$ plate width, $m$

$\alpha$ thermal expansion coefficient, $\frac{1}{K}$

$\beta$ incident shock angle

$\delta x_s$ panel deflection, $m$

$\Delta_{BL}$ boundary layer thickness, $m$

$\delta$ nodal displacement, $m$

$\Phi$ phase angle, deg

$\gamma$ adiabatic exponent

$\lambda$ nondimensional dynamic pressure

$\mu_s$ panel mass ratio

$\mu$ dynamic viscosity of fluid, $\frac{kg}{m \cdot s}$

$\nu_p$ Poisson’s ratio

$\nu$ kinematic viscosity of fluid, $\frac{m^2}{s}$

$\omega$ molecular weight

$\rho_f$ fluid density, $\frac{kg}{m^3}$

$\rho_s$ plate density, $\frac{kg}{m^3}$

$\tau_w$ wall shear stress, $\frac{N}{m^2}$
1.1 Fluid-Structure Interactions

Recent trends in air vehicle technology development include a renewed interest in supersonic flight vehicles and the use of lightweight aero-structures in their construction. Supersonic flight trajectories may involve segments of transonic flow regimes with mixed supersonic/subsonic flow and shock-boundary layer interactions (SBLI) over the surface of the vehicle. For such flows, the aerodynamic forces on the vehicle can generate strong elastic stresses on its structural members. If these stresses produce changes to the geometry of the vehicle’s outer surfaces, then the fluid flow path is altered and thus the aerodynamic forces are modified as well. This process results in complex coupling of fluid-structure interactions (FSI), commonly referred to as aeroelasticity. Collar’s aeroelastic triangle, shown in Figure 1.1, provides a visualization of the interaction of the aerodynamic, elastic, and inertial forces in a dynamic system as related to aeroelasticity [7].

1.1.1 Canonical Aerospace Applications of FSI
Stability characteristics of a dynamic system change with system parameters. A system undergoing FSI can be strongly influenced by dynamic pressure, Mach number, Reynolds number, presence of nonlinearities in fluid (shocks, flow separation, etc.) and structure (large deformation). As the damping characteristics of the system change with increasing dynamic pressure (with other parameter values held fixed), the system may settle into self-sustained oscillations due to effective zero damping in the system response, resulting in the flutter phenomenon. This can lead to material fatigue and premature failure of the structural members. Hence, it is important to study and understand the impact of various parameters on the system stability characteristics.

Experiments [11, 27] and numerical simulations have been used to predict and/or improve our understanding of fluid-structure interactions in transonic flows in order to assist in the design of next generation aircraft. The analysis of these complex interactions typically requires time-accurate resolution of both the fluid and structural responses. Various partitioned and monolithic FSI approaches have been developed to assist with this analysis.
The partitioned approach will typically involve establishing some form of communication of loads and displacements between established (or purpose-built) standalone fluid and structural dynamic solvers. In contrast, the monolithic solver approach requires simultaneous discretization of the governing equations for both the fluid and structural disciplines with coupled Jacobian terms at the FSI boundary [23].

Piston theory, which has been widely used to inexpensively predict the fluid forces in a FSI simulation, estimates the aerodynamic pressure using an analytic model based on the structural deflection and normal velocity [14, 24]. In recent studies, Ganji and Dowell [14] developed an aerodynamic damping enhanced piston theory to predict flutter of a semi-infinite panel for low supersonic range \((M \text{ from 1 to } \sqrt{2})\). The study reported that the enhanced model can predict single mode flutter expected in low \(M\) ranges, which cannot be predicted by the classical piston theory. Brouwer and McNamara [4] further enhanced the piston theory with a quasi-steady flow assumption wherein the linear perturbations in the surface pressure were added to a local pressure obtained from a steady CFD solution. The model was applied for stationary and oscillating shock impingement in two- and three-dimensional flows, and the results were compared against benchmark Euler and URANS simulations. The proposed approach was able to capture shock-induced limit cycle oscillations accurately. However, the accuracy degraded for increasing modal activity. The performance degradation was attributed to the influence of unsteady viscous forces.

Several studies have coupled the inviscid Euler equation solver for aerodynamics with modal structural dynamics solver [8]. Lee-Rausch [19] applied this model to examine the transonic flutter boundaries of an AGARD 445.6 airfoil. The results compared well with
the experiments for $M < 1$, but also predicted a premature rise in flutter boundary for $M > 1$. Chen et al. [6] applied the model for three-dimensional transonic wing flutter for the AGARD 445.6 configuration and reported reasonable agreements with the experiment. Visbal [32] coupled Euler equations for fluid flow with the von Karman plate equations for structural dynamics. The nonlinear von Karman strain, which is a simplification of Green-Lagrange strain for thin bending structures, provides the coupling between in-plane and moderately large transverse deflections. The model was applied for simulation of semi-infinite panel flutter due to shock impingement. The study showed, for the first time, that a shock of sufficient strength could induce bifurcation at a significantly lower dynamic pressure than that of the standard panel flutter. Shishaeva, Vedeneev, and Aksenov [30] used Mindlin plate theory combined with nonlinear von Karman strain. They applied the solver for finite panel flutter simulations. The results demonstrated existence of non-periodic oscillations for $1.33 < M < 1.42$, and coupled flutter mode for $M = 1.82$. Bhatia and Beran [1] coupled a compressible Euler flow solution (linearized about the mean) with structural dynamics solver to formulate a linearized stability eigenvalue problem. The formulation was used to evaluate flutter instabilities arising from fluid-structure interaction obtained for semi-infinite and square panels for a range of subsonic and supersonic flow conditions. The study reported that for both the semi-infinite and square plates, the high-frequency flutter modes become critical in a very narrow range of the dynamic pressures for low supersonic flows, whereas classic supersonic flutter mode behavior is observed for higher Mach numbers. Boyer et al. [3] performed simulations for shock impingement induced dynamics of a
square panel using coupled Euler and von Karman plate solvers. They reported that flutter amplitude and frequency increased significantly with increases in shock-strength.

Other studies have utilized coupled viscous Navier-Stokes solvers with structural solvers to study FSI for laminar and turbulent conditions. Gordnier and Visbal [16] developed a three-dimensional aeroelastic solver by strongly coupling the full Navier-Stokes equations with the von Karman plate equations. This study provides an in-depth validation of semi-infinite transonic panel flutter. For supersonic cases that include viscous effects, the laminar boundary layer was shown to delay bifurcation, resulting in a higher critical dynamic pressure and lower frequency. Much like in the inviscid case, subsonic flow conditions were observed to exhibit two types of behavior, one with a downward deflection of the panel, and the other with the upper deflection becoming unstable due to shock formation across the surface of the panel. Gordnier and Visbal [15] expanded upon this work by applying the computational methodology to a square panel with prescribed laminar and turbulent boundary layers, wherein the Baldwin-Lomax turbulence model was used for the unseparated flows. Their results echoed the conclusion of the previous study; that the presence of the boundary layer delayed the onset of panel flutter. With the addition of turbulence, it was observed that the effect was much less pronounced for the turbulent boundary layer profiles. Li et al. [20] applied the model for analysis of shock impingement induced panel flutter for inviscid and laminar boundary layer flows. They concluded that an aero-elastically tailored flexible panel could potentially be used as a means of passive flow control.
Cavagna, Quaranta, and Mantegazza [5] extended the model developed by Chen et al. [6] to include viscous effects by coupling RANS fluid solver with modal structural solver. The solver was validated for aeroelastic stability evaluation of an AGARD 445.6. Ozcatalbas, Acar and Uslu [26] performed an aeroelastic analysis of AGARD 445.6 using weakly coupled commercially available fluid and structural solvers and a weak coupling method along with the Spalart-Allmaras turbulence model for viscous effects. The flutter speeds and frequencies in the transonic regime were determined to be in good agreement with experiment and numerical studies. Wang [33] also utilized the loosely coupled Spalart-Allmaras (RANS) model with the second order Bernoulli beam element model to simulate static aeroelastic behavior of the ONERA M6 wing over a range of angles of attack.

Im, Chen and Zha [18] and Gan and Zha [13] performed simulation of supersonic flutter boundaries on AGARD 445.6 using the full Navier-Stokes equations strongly coupled with the modal approach for structural dynamics. They reported that prior URANS studies that capture the flutter boundary of subsonic flows compare fairly well with experiment; however, they performed poorly in predicting flutter with supersonic inflow conditions involving shock boundary layer interaction. The study demonstrated that using hybrid RANS/LES with delayed detached eddy simulation (DDES) improved the flutter predictions. Ostoich et al. [25] investigated the interaction of thin, metallic panel and laminar and turbulent boundary layer using DNS which was coupled with a finite-element structure solver. Their results showed that the panel flutter generated oscillating compression waves which significantly augmented the turbulent structures in the near-wall region. Vedeneev [31] investigated the effect of boundary layer on plate stability during panel flutter using
a laminar Navier-Stokes solver coupled with von Karman structural solver. They reported that flutter instabilities can be categorized into subsonic (multiple-mode) and supersonic (single-mode) modes. The former is activated for accelerating flows (for a generalized convex boundary layer), whereas the latter is active for the decelerating flows (for a generalized boundary layer profile with an inflection point). Shinde, McNamara, and Gaitonde [29] studied flutter induced by an oblique shock impinging on a three-dimensional transitional and turbulent boundary layer developing over a flexible panel using a coupled Navier-Stokes and von Karman solvers. The simulations were performed using DNS. They reported that the coupled SBLI and panel flutter enhanced the transition to turbulence and resulted in an unsteady separation region which then served as a source of acoustic radiation.

This review of the literature shows that the complex flow physics involving transitional/turbulent boundary layers and shock impingement remains an open area of research. The modeling of coupled fluid-structure interactions is steadily trending towards high-fidelity simulations using coupled LES/DES/DNS fluid and finite-element structural solvers to abridge the existing knowledge gap.

Such a high-fidelity FSI solver capability has recently been developed by the authors, and the validation of this capability is described in this report. This capability has been implemented by incorporating the structural modeling capabilities of the MAST (Multidisciplinary-design Adaptivity and Sensitivity Toolkit) finite-element library [2] into a tightly coupled FSI module for the Loci-Chem flow solver [21].
1.2 Objectives and Approach

The objective of the research is two-fold: (a) support FSI code development through validation for semi-infinite panel deformation test cases, including shock impingement; and (b) validation and exploratory study to evaluate the effect of boundary layer on panel deformation behavior.

To achieve objective (a):

(1) Inviscid simulations were performed for a semi-infinite panel subject to uniform subsonic ($M = 0.9–1.0$) and supersonic ($M = 1.0–2.0$) flows, the effect of flow velocity (or dynamic pressure) on the bifurcation location was studied, and the steady-state equilibrium/limit-cycle oscillation amplitudes and associated frequencies were validated against the benchmark results of [15] and [1].

(2) Inviscid simulations were also performed for a semi-infinite panel subject to an impinging oblique shock wave of varying shock strength ($P_3/P_1 = 1.2, 1.4, 1.8$). For weak shocks ($P_3/P_1 = 1.2$), steady-state equilibrium panel deformation and surface pressure ratios were validated against [32]. For strong shock strengths ($P_3/P_1 = 1.8$), the effect of increasing flow velocity was studied, and the bifurcation location predictions was validated against [32]. For medium shocks ($P_3/P_1 = 1.4$), panel deformation and surface pressure ratio during LCO was validated against [20]; and the effect of the shock strength on the bifurcation location and LCO behavior is discussed.

To achieve objective (b):
(1) Laminar simulations were performed for panel subject to an impinging oblique shock wave impingement with $P_3/P_1 = 1.4$ for two boundary layer thicknesses and a fixed laminar Reynolds number. Limit-cycle panel deformations, surface pressure ratios, and frequency spectra were validated against [20];

(2) Laminar and turbulent simulations were performed for oblique shock wave impingement with $P_3/P_1 = 1.4$ for a wide range of flow velocities (or dynamic pressures), and corresponding laminar and turbulent solutions were explored for the same range. Bifurcation location, panel unsteadiness and flow separation patterns were analyzed to identify the similarities and differences between inviscid, laminar and turbulent predictions.

In addition, a comprehensive grid refinement analysis was also performed for each of the cases to examine the effect of increasing levels of resolution on the bifurcation point, steady-state equilibrium/LCO amplitude, and flutter frequency. A grid was chosen which exhibits a balance of solution accuracy and computational efficiency. Table 1.1 contains an overview of both validation cases and exploratory studies performed in this research.
Table 1.1: Full Simulation Table

<table>
<thead>
<tr>
<th>Case</th>
<th>Flow Properties, air</th>
<th>Structure Properties</th>
<th>Grid size&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Time step size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$\lambda$</td>
<td>$\rho_2/\rho_1$</td>
<td>$\delta/L$</td>
</tr>
<tr>
<td>Uniform Flow</td>
<td>0.4 – 0.95</td>
<td>30 – 3000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.0 – 2.0</td>
<td>2 – 880</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Oblique shock</td>
<td>Inviscid</td>
<td>2</td>
<td>200 and 875</td>
<td>1.2, 1.4 and 1.8</td>
</tr>
<tr>
<td></td>
<td>Laminar</td>
<td>875</td>
<td>1.4</td>
<td>0.0156, 0.0262</td>
</tr>
<tr>
<td>Oblique shock</td>
<td>Inviscid</td>
<td>2</td>
<td>100 – 800</td>
<td>1.4 and 1.8</td>
</tr>
<tr>
<td></td>
<td>Laminar</td>
<td>100 – 875</td>
<td>1.4 and 1.8</td>
<td>0.0156</td>
</tr>
<tr>
<td></td>
<td>Turbulent</td>
<td>100 – 875</td>
<td>1.4 and 1.8</td>
<td>0.0156</td>
</tr>
</tbody>
</table>

<sup>a</sup>Multiple grid sizes indicate convergence study where bold values indicate selected grid size.
CHAPTER 2

PROBLEM OVERVIEW

2.1 Flutter

Flutter is an aeroelastic instability phenomenon characterized by self-excited oscillations that arise as a direct response to the interplay between the dynamic pressure fluctuations in the flow field and the deformation of the structure. As dynamic pressure is increased, damping characteristics of the coupled system can change such that the energy collected from the freestream cannot be properly attenuated; thus, the structure develops self-sustained oscillations. This can lead to premature fatigue and failure of the structural members. Extensive reviews by Dowell [10] and Mei [24] on the topic are well known in the field of flutter and provide more detail on the physical qualities of the phenomenon. Although, as flight structures trend toward hypersonics, it is important to obtain a deeper understanding of the underlying flow mechanisms that drive these coupled responses. The following provides an overview of the steps taken to set up and validate a coupled fluid/structural numerical solver for fluttering panels.
2.1.1 Problem Description

The generalized problem description is given in Figure 2.1 where $L$ is the length of the panel, $M$ is the freestream Mach number, and $P_1$ is the freestream pressure in Pa. The flow is initialized as uniform across the entirety of the domain.

![Figure 2.1](image)

**Figure 2.1**

Uniform flow problem description.

Parameters defining the coupled response of the system are determined by the Mach number $M$, nondimensional dynamic pressure parameter $\lambda$, and the mass ratio $\mu_s$. The mass ratio, given in Equation (2.1), is the ratio of the mass of the fluid to the mass of the structure and $\lambda$, shown in Equation (2.3), considers critical fluid and structural properties and relates them to a set of freestream parameters, where $\rho_f$ is the freestream fluid density, $u$ is the freestream velocity, $L$ is the length of the panel, and $D$ is the flexural rigidity. In this analysis, $\mu_s$ is fixed so as to maintain a constant mass ratio such that Equation (2.1) can be rearranged to solve for $\rho_f$, as seen in Equation (2.2). The flexural rigidity is a parameter that depends solely on material properties, as denoted in Equation (2.4), where $E_s$ is the Young’s modulus, $H$ is the panel thickness, and $\nu_p$ is Poisson’s ratio.
\[ \mu_s = \frac{\rho_f L}{\rho_s H} \]  

(2.1)

\[ \rho_f = \frac{\mu_s \rho_s H}{L} \]  

(2.2)

\[ \lambda = \frac{\rho_f u^2 L^3}{D} \]  

(2.3)

\[ D = \frac{E_s H^3}{12(1 - \nu^2)} \]  

(2.4)

Alternatively, for a specified \( \lambda \), material properties, and fluid constants (\( \gamma = 1.4 \) and \( R_s = 287.80 \text{ J/kgK} \) for air), one can derive the set of freestream parameters, namely \( P_f \) and \( T_f \), to populate the initial and boundary conditions at the domain inlet. Those derivations are given in Equations (2.5) and (2.6) for \( P_f \) and \( T_f \), respectively. This procedure that is used to initialize the domain with proper fluid properties prior to numerical simulation.

\[ P_f = \frac{\lambda D}{M^2 \gamma L^3} \]  

(2.5)

\[ T_f = \frac{\lambda D}{\rho_f M^2 \gamma R_s L^3} \]  

(2.6)

The speed of sound of the freestream, and the corresponding freestream velocity is defined by the relations of Equations (2.7) and (2.8), respectively.

\[ a = \sqrt{\gamma R_s T_f} \]  

(2.7)
\[ u = Ma \] (2.8)

The cavity pressure on the underside of the panel, \( P_0 \), is set to be equivalent to the pressure of the freestream, \( P_1 \). This initialization process creates a balance of pressures below and above the panel such that the flow cannot directly perturb the motion of the panel. Therefore, an initial velocity perturbation is defined at the panel mid-chord in order to initialize the deformation.

2.2 Shock Impingement

Due to the nature of compressible flows, especially in flight environments where aircraft may reach supersonic speeds, sonic shock waves can develop which often impinge on vehicle exteriors or engine interiors. These interactions deliver large, localized loading which can further contribute to dynamic instabilities that may already arise from the flow (i.e. in the case of panel flutter). For that reason, shock-boundary layer interactions (SBLI or SWBLI) have been heavily studied since the early 1940’s [9].

2.2.1 Problem Description

Figure 2.2 contains the generalized problem description for shock impingement. This case is a simple extension of uniform flow to include an incoming and reflected oblique shock wave impinging along the panel mid-chord. A large pressure gradient is formed once across the incoming shock and again as the shock is reflected. The resulting system is, therefore, comprised of three zones each with a prescribed pressure, \( P_1 \), \( P_2 \), and \( P_3 \), determined by the strength of the shock wave.
The shock strength is often identified by the ratio of the reflected shock pressure to the incoming freestream pressure, $P_3/P_1$. Once the freestream zone (zone 1) is populated with parameters as defined by $\lambda$ and other relevant structure parameters as in the flutter case, geometric oblique shock wave relations are used to obtain the remaining zonal properties [17]. There exists an additional geometric parameter, the incident shock angle $\beta$, that is closely associated with the pressure ratio and can independently define the zonal relationships of the incoming and reflected shock wave. Table 2.1 contains the shock angle and corresponding pressure ratios used in this study.

Table 2.1: Pressure Ratio/$\beta$ Relationship

<table>
<thead>
<tr>
<th>$P_3/P_1$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>N/A</td>
</tr>
<tr>
<td>1.2</td>
<td>31.349°</td>
</tr>
<tr>
<td>1.4</td>
<td>32.584°</td>
</tr>
<tr>
<td>1.8</td>
<td>34.799°</td>
</tr>
</tbody>
</table>
For a given freestream Mach number, \( M \), pressure, \( P \), specific heat ratio, \( \gamma \), and incident shock angle, \( \beta \), the pressure and density aft the incoming shock (i.e. in zone 2) is revealed by Equation (2.9) and Equation (2.10). Across the large pressure gradient, the velocity will not maintain a purely horizontal directionality; the direction of the outgoing flow is defined by the so-called deflection angle of the shock wave. For the incoming shock, the deflection angle is defined by Equation (2.11), from which the Mach number can be calculated, as seen in Equation (2.12).

\[
P_2 = P_1 \frac{2\gamma M_1^2 \sin^2(\beta) - (\gamma - 1)}{\gamma + 1} \quad (2.9)
\]

\[
\rho_2 = \rho_1 \frac{(\gamma + 1)M_1^2 \sin^2(\beta)}{(\gamma - 1)M_1^2 \sin^2(\beta) + 2} \quad (2.10)
\]

\[
\theta = \arctan \left( \frac{2}{\tan(\beta)} \frac{M_1^2 \sin^2(\beta) - 1}{M_1^2 (\gamma + \cos(2\beta)) + 2} \right) \quad (2.11)
\]

\[
M_2 = \frac{1}{\sin(\beta - \theta)} \sqrt{\frac{(\gamma - 1)M_1^2 \sin^2(\beta) + 2}{2\gamma M_1^2 \sin^2(\beta) - (\gamma - 1)}} \quad (2.12)
\]

Parameters of zone 3 are found in much the same way as that of zone 2 by replacing the coefficients in the preceding equations and adjusting angles to account for incoming, non-horizontal vectors. The cavity pressure is considered to be the mean pressure along the panel length as established by inviscid oblique shock reflection theory and is given in Equation (2.13), where \( X \) is the horizontal distance to the impingement location. The prescribed cavity pressure remains for the duration of the simulation. For all cases in the
present study, the shock is impinging at the mid-chord, or \( X/L = 0.5 \). The equation therefore simplifies to the average of \( P_1 \) and \( P_3 \) (i.e. \( P_0 = 0.5(P_1 + P_3) \)).

\[
P_0 = (X/L)P_1 + [1 - (X/L)]P_3
\]

(2.13)

2.3 Identification of Bifurcation Points for Coupled Fluid-Structure Response

The results provided in the following sections present a detailed investigation of the coupled fluid-structure response of a flexible panel in either uniform flow or in flow with an impinging oblique shock. Nonlinear dynamical systems, such as the fluid-structure interaction system studied here, exhibit a rich set of responses. Of particular interest here are the fixed-point, bifurcation point and limit-cycle oscillation (LCO). A fixed-point identifies a static equilibrium condition which could be stable or unstable (Figure 2.3). Infinitesimal perturbations at a stable fixed-point lead to a damped oscillatory response such that the system returns to the fixed-point. On the other hand, perturbations about an unstable fixed-point cause the response to diverge away from this point. The locus of fixed-points with an increasing system parameter (dynamic pressure in the present case) creates a branch. A bifurcation point identifies the point on this branch where the nature of fixed-point changes from stable to unstable or vice-a-versa. New branches can emerge from bifurcation points. A branch of unstable fixed points can be surrounded by limit-cycle oscillations which is a nonlinear time-periodic response of the system.

The following numerical investigations study the impact of Mach number and dynamic pressure on the nature of response of the coupled system. For a given Mach number the
A schematic showing the stable (green) and unstable (red) fixed-point branches for a nonlinear dynamical system. \( p \) is a system parameter and \( y \) is a component of the system state vector.

time-accurate response of the system is recorded for multiple values of dynamic pressures. For investigation under a uniform flow (without shocks), a small initial velocity of ±0.01 m/s is specified at the midpoint of the panel to introduce a small perturbation to the panel geometry. The shock impingement cases do not require any such perturbation since the large pressure gradient of the incident and reflected shocks are sufficient to initiate structural deformations in the panel. The time history is used to qualify the response as a stable fixed-point or limit-cycle oscillation. A bracketing approach is used to identify the bifurcation dynamic pressure where a stable branch becomes unstable and new branches (of fixed point or limit-cycle oscillation) appear. A frequency response analysis of the time history of a limit-cycle oscillation response is used to identify the dominant frequencies in the panel dynamics.
CHAPTER 3

COMPUTATIONAL METHODOLOGY

3.1 Fluid Solver

Loci-Chem [21] is a multi-species, chemically reacting, flow solver that has been extensively validated for highly energetic flows found in rocket plumes. The governing equations for the fluid flow are the mass-weighted, filtered, compressible Navier-Stokes equations for which the mass, momentum, and energy equations are given in Equations (3.1, 3.2, and 3.3).

\[
\frac{\partial}{\partial t}(\hat{\rho}_f) + \frac{\partial}{\partial x_j}(\hat{\rho}_f \hat{u}_j) = 0 \tag{3.1}
\]

\[
\frac{\partial}{\partial t}(\hat{\rho}_f \hat{u}_i) + \frac{\partial}{\partial x_j}(\hat{\rho}_f \hat{u}_i \hat{u}_j) = -\frac{\partial \hat{P}}{\partial x_i} + \frac{\partial}{\partial x_j}(\sigma_{ij}) \tag{3.2}
\]

\[
\frac{\partial}{\partial t}(\hat{\rho}_f \hat{E}) + \frac{\partial}{\partial x_j}(\hat{\rho}_f \hat{u}_j \hat{H}) = \frac{\partial}{\partial x_j}(k \frac{\partial \hat{T}}{\partial x_j} + \hat{u}_i \sigma_{ij}) \tag{3.3}
\]

In the above equations, \(\hat{\rho}_f\) is the filtered fluid density, \(\hat{u}_i = \rho_f \hat{u}_i / \hat{\rho}_f\) are the mass-weighted filtered velocity components, \(\hat{P}\) is the fluid pressure, \(\hat{E}\) is the total energy, \(\hat{H}\) is the total enthalpy, \(\hat{T}\) is the fluid temperature, and \(\sigma_{ij}\) are the viscous stresses. In the present
work, the fluid is assumed to be a thermally perfect gas; that is, the fluid mixture obeys the following simple thermal equations of state (EoS).

\[ \hat{P} = \hat{\rho} f R_s \tilde{T} \]  

\[ \tilde{E} = c_v \tilde{T} + \frac{1}{2} \tilde{u}_k \tilde{u}_k \]  

\[ \tilde{H} = c_p \tilde{T} + \frac{1}{2} \tilde{u}_k \tilde{u}_k \]  

The specific gas constant is \( R_s = 287.8 \text{m}^2/\text{s}^2 - \text{K} \), and the total viscous stress tensor is defined in Equation (3.7).

\[ \sigma_{ij} = \mu \left[ \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} \right] \]  

With the thermally perfect gas assumption, the fluid properties \( k, \mu \) and specific heat capacities \( c_p, c_v \) are specified as a function of \( \tilde{T} \). For the present work, these quantities are interpolated from internal equation of state tables for single species air with no reactions. The governing equations are discretized using a cell-based finite-volume approach that supports generalized grids. The solver provides several numerical schemes from 2nd to 4th order accurate, as well as URANS and hybrid RANS/LES turbulence models. For the results presented in this report, the simulations were performed using 2nd order accurate spatial fluxes with along with a 2nd order 3 point backward time integration scheme with a specified global timestep. The volume fluid mesh is adapted in response to each surface
deformation update according to the inverse distance weighted (IDW) method described in [22].

3.1.1 Structural Solver

The Multidisciplinary-design Adaptivity and Sensitivity Toolkit (MAST) is used as the structural solver [2]. MAST has been extensively validated for transonic flutter analysis and the design of thermally stressed structures [1]. The structural finite-element discretization in MAST is based on Bernoulli beam theory for semi-infinite panels and Mindlin plate for finite panels. The nonlinear influence of stretching due to bending is modeled with the von Karman strain, which provides coupling between the in-plane and transverse displacements. The variational statement for a semi-infinite panel is written as: find $x_s$ such that for all $\delta x_s$

$$B(\delta x_s, x_s) = f^a \left( \delta x_s, x_s, \frac{\partial x_s}{\partial t} \right)$$  \hspace{1cm} (3.8)

$$B(\delta x_s, x_s) = \int_{-h/2}^{h/2} \int_0^L \left[ \rho \delta x_s^T \frac{\partial^2 x_s}{\partial t^2} + \epsilon_{xx}^\Delta(x_s, \delta x_s)^T \sigma_{xx}(x_s) \right] dx dy$$  \hspace{1cm} (3.9)

$$f^a \left( \delta x_s, x_s, \frac{\partial x_s}{\partial t} \right) = \int_0^L \left[ \begin{array}{c} 0 \\ \tau_f(x_s, \frac{\partial x_s}{\partial t}) \end{array} \right] dx$$  \hspace{1cm} (3.10)

Where, $\tau_f$ is the normal fluid stresses acting on the structural element, $\epsilon_{xx}(x_s)$ is the longitudinal (along $x$-axis) nonlinear strain, $\epsilon_{xx}^\Delta(x_s, \delta x_s)$ is the linearized strain due to a
small perturbation $\delta x_s$, and $\sigma_{xx}(x_s)$ is the stress due to $\epsilon_{xx}(x_s)$. A similar formulation is defined for a structure modeled with plate elements, and details can be found in [1]. The above equation is represented in semi-discrete matrix-vector form as:

$$
M^s \frac{d^2 X}{dt^2} + F^I(X) - F^a \left( X, \frac{dX}{dt} \right) = 0
$$

(3.11)

The matrix, $M^s$ and vectors $F^I$, $F^a$ are related to the variational statement as:

$$
\delta X^T M^s \frac{d^2 X}{dt^2} = \int_{-h/2}^{h/2} \int_0^L \rho \delta x_s^T \frac{\partial^2 x_s}{\partial t^2} dx dy
$$

(3.12)

$$
\delta X^T F^I(X) = B(\delta x_s, x_s)
$$

(3.13)

$$
\delta X^T F^a \left( X, \frac{dX}{dt} \right) = f^a \left( \delta x_s, x_s, \frac{\partial x_s}{\partial t} \right)
$$

(3.14)

### 3.1.2 Coupling Algorithm

The fluid and structural dynamics solvers are coupled through a virtual layer of rigid structural elements which permit a two-way, work-conservative, mapping of fluid forces and displacements between the fluid and structural nodes at the shared interface as reflected in Figure 3.1. In this method of FSI coupling, each fluid node on the fluid domain’s FSI boundary forms either a tetrahedron or a pyramid with one face on the structural domain’s FSI boundary. This creates a layer of scaffold elements on the FSI boundary between the
fluid and structural domains. These scaffold elements form a static system of equations described by:

\[ \mathbf{K} \delta \mathbf{X} = \mathbf{f} \]  \hspace{1cm} (3.15)

where, \( \mathbf{K} \) is the stiffness matrix, \( \delta \mathbf{X} \) is the displacement vector, and \( \mathbf{f} \) is the force vector.

![A tetrahedral element used for coupling fluid and structure.](image)

**Figure 3.1**

A tetrahedral element used for coupling fluid and structure.

When the structural domain has a quadrilateral element, a pyramid is used as a scaffold element such that the fluid node is contained within its volume. Then, similar procedure can be used to transfer the forces from the fluid nodes to the structural nodes, and the displacements from the structural nodes to the fluid nodes.

With this transfer scheme, the fluid and structural solvers are coupled using the conventional serial staggered (CSS) procedure where the fluid solver’s FSI boundary is lagged behind the structural solver. The same time step is used for both solvers and the fluid forces
and structural displacements are exchanged once per time step. A more detailed explanation of the coupling algorithm can be found in the works of Schemmel et al. [28] and Zope et al. [34].

3.2 Domain, Grid, and Boundary Conditions

Throughout this study, a number of grids were utilized depending on whether the present case was uniform flow or shock impingement. Some validation cases required specific grid dimensions and constraints, and those were adjusted accordingly. All solutions consider a 2D flow field where a 3D domain is constrained to be a single cell wide in the z-direction.

3.2.1 Inviscid Uniform Flow

The benchmark panel flutter simulations in 2D uniform inviscid flow utilize a semi-infinite panel of length \( L = 0.3m \) and thickness \( H = 0.0015m \). The panel is located at \( y = 0 \) and spans \( x = [1.5, 1.8]m \). The panel structure is modeled as a beam using 30 elements. The fluid domain shown in Figure 3.2 spans \( x = [0, 3.3]m \) and \( y = [0, 5]m \). The fluid domain is discretized using an unstructured mesh of 22,700 cells. As a side note on the domain shape, as the panel begins to deform, pressure fluctuations can sometimes form atop the panel which can propagate up and toward the right edge of the domain where the pressure waves may exit more than one plane simultaneously. This unique case was observed to occur in a handful of cases, so prevent any numerical abnormalities, the outflow boundary was extended such that all propagating waves will pass through it.
3.2.2 Inviscid Shock Impingement

The fluid and structure domains for the 2D inviscid shock impingement cases are very similar to that of the 2D uniform inviscid flow cases with the exception that the mesh has been refined in regions of interest (near the panel and along the incident and reflected shocks) as shown in Figure 3.3. The panel again has a length of $L = 0.3m$, but with a thickness of only $H = 0.0006m$. (These dimensions are consistent with that of [32] and [20].) The panel structure modeled by a beam with 70 elements. The fluid domain consists 54,000 grid cells.

3.2.3 Viscous Shock Impingement

For the case of an oblique shock impinging on a viscous boundary layer, a fully structured mesh is used which spans $x = [1.32, 3.3]m$ and $y = [0, 1.5]m$. The domain is shown
in Figure 3.4. The domain is discretized into a block of $231 \times 151$ quad elements. The panel uses 50 beam elements and is located at $x = [1.5, 1.8] m$ and $y = 0 m$ and has a thickness of $H = 0.0006 m$. Since the grid resolution nearest the panel greatly outweighs the grid resolution away from the panel, pressure fluctuation are quickly dissipated which prevents any numerical abnormalities that may be generated by waves interacting with more than one boundary. Therefore, the rounded farfield boundary from Subsection (3.2.1) and Subsection (3.2.2) was revised such that the mesh is rectangular.

To achieve comparable Reynolds number to existing numerical studies, a constant viscosity transport model was utilized. This allows for a specification of a fluid thermal conductivity $k_{cond}$ and dynamic viscosity $\mu$ such that a fixed Reynolds number is maintained in the freestream. The fluid thermal conductivity calculation is given in Equation (3.16)
where the fluid density $\rho_f$ is defined by Equation (2.2) and the thermal expansion coefficient $\alpha$ is given by Equation (3.17). The equation for the kinematic viscosity, $\nu$, can be found in equation (3.18) where $u$ is the freestream velocity, $H$ is the panel length, and $Re$ is the specified Reynolds number. The heat capacity for air is $c_p = 1006 \text{ J/kg K}$ and the Prandtl number is $Pr = 0.72$. Finally, the dynamic viscosity is given by Equation (3.19).

$$k_{cond} = \rho_f c_p \alpha$$  \hspace{3cm} (3.16)

$$\alpha = \nu/Pr$$  \hspace{3cm} (3.17)

$$\nu = u H/Re$$  \hspace{3cm} (3.18)

$$\mu = \nu \rho_f$$  \hspace{3cm} (3.19)
One issue associated with the validation of the present coupled solver for viscous flows is the difficulty in producing boundary layer properties that are consistent with published numerical solutions. Several benchmarks present boundary layer properties only in terms of boundary layer thickness. Other relevant parameters such as the displacement and momentum thickness at specified locations would allow for more flexibility in initializing the boundary layer. To ensure the most appropriate representation possible, the domain was constructed such that the distance from the domain inlet to the panel leading edge was the same as that of the numerical study, and the boundary layer was permitted to grow naturally using a viscous wall boundary condition. This method, combined with the previously described constant viscosity transport model, provided a boundary layer which was consistent in both boundary layer thickness and Reynolds number. For thicker boundary layers, the Blasius solution was initialized in the initial and boundary conditions throughout the domain, where for a specified y-range, the x-velocities where damped according to the location of the node in the boundary layer.

3.3 Validation Methodology

Validation focused on the prediction of dynamic instabilities arising from the inviscid fluid-structure coupling in the absence of a shock in 2D (i.e. uniform flow). The shock-boundary layer interaction cases focused on the prediction of structural responses in 2D due to an oblique shock impinging on a plate in inviscid flow as well as for viscous flow
with a laminar boundary layer of prescribed thickness and Reynolds number as outlined in Table 2.1.

The results for panel divergence for subsonic flows and panel flutter and LCO for supersonic flows with respect to dynamic pressure variations were compared with [15, 16]. The predictions of panel flutter modes and bifurcation points were validated against [16] and [1]. For the oblique shock interaction cases, the predictions of flutter amplitude and panel divergence for inviscid flows were compared with [32]. The effects of the viscous boundary layer on the panel flutter were compared with [20].

To perform a validation study, key structure and fluid parameters were typically given within each reference for proper replication of results. For uniform flow cases, the key parameters given for the fluid were Mach number and nondimensional dynamic pressure, while mass ratio and Poisson’s ratio were typically given for the structure. These parameters were equally important for oblique shock impingement, only with the addition of the thickness to length ratio of the panel, or \( H/L \); vastly different behaviors were initially observed for a different \( H/L \) than was reported in the literature while the Mach number and dynamic pressure were held constant. Structure parameters such as Young’s modulus and density significantly alter freestream properties \((P, T, u, a)\), but as long as the key structure and fluid parameters remain the same, the same representative behavior is expected.

The dynamic pressure, Mach number, and mass ratio given in the benchmark description was passed through equations 2.1-2.6 to calculate the inflow pressure and temperature of the freestream. These properties then use ideal gas relations to obtain other relevant freestream parameters and populate the initial and boundary conditions of the fluid sim-
ulation. Immediately following, a proper time step was chosen to ensure the oscillatory nature of the response was adequately captured. Proper output intervals were also set such that slices of the volume solution could be examined at key points in the response. A typical technique of examining flutter is tracking the vertical displacement of the mid-chord ($X/L = 0.5$) and 3/4-chord ($X/L = 0.75$) over time. Probes were also placed at these locations throughout each simulation to observe the flutter patterns. The simulation was then allowed to proceed to completion.
CHAPTER 4
VALIDATION RESULTS

4.1 Uniform Flow

This section presents results for the response of a semi-infinite and square panel under uniform inviscid flow. Details about the bifurcation analysis at a subsonic Mach number ($M = 0.9$) are presented in 4.1.1, where the panel is shown to exhibit divergence instability. The bifurcation analysis at supersonic Mach numbers ($M = 1.2, 1.414, 1.6, 2.0$) are discussed in Sections 4.1.2 - 4.1.5, where bifurcation and limit-cycle oscillations are characterized along with their frequency response. The nondimensional dynamic pressure and frequency are computed at multiple Mach numbers and compared against Bhatia and Beran [1] in section 4.1.6.

4.1.1 Semi-Infinite Panel at $M = 0.9$

The panel response to a small perturbation is simulated for increasing dynamic pressures at $M = 0.9$. The amplitude of the mid-point displacement of the panel in Figure 4.1 shows that the bifurcation point occurs at approximately $\lambda = 15.55$. Below this value of $\lambda$ the response due to an initial perturbation is damped out and the panel returns to a
flat equilibrium configuration (Figure 4.2(a)) with the flow field becoming uniform. This indicates that the system state at zero-displacement with uniform flow is a stable fixed point. An initial perturbation above the bifurcation point causes the panel to settle into a different equilibrium configuration with either a positive or negative panel displacement (Figure 4.2(b)). Without an initial perturbation the panel would continue to stay in an undeformed configuration, indicating that the zero-displacement branch is now unstable since a small perturbation causes the panel to diverge away from this point and settling into a nearby fixed point. This bifurcation is called divergence in aeroelastic literature which is a linearized static instability at zero frequency.

![Graph](attachment:image.png)

Figure 4.1

Nondimensional mid-chord displacement amplitude vs. dynamic pressure for $M = 0.9$.

Two new stable branches emerge from the bifurcation point (Figure 4.1) corresponding to stable equilibrium configurations with positive and negative panel displacements. In the following discussion these are referred to as convex (into the flow) and concave (away from the flow) shapes, respectively. The panel deflection on both of these branches increases
Figure 4.2

Time history of mid-panel displacement for a stable fixed point at (a) $\lambda = 15$ and (b) $\lambda = 17$.

with $\lambda$. However, the nonlinear stiffening effects due to the von Karman strain reduces the rate of growth of displacement with increasing $\lambda$. For flow up to $\lambda = 700$, the panel deflection converges to either concave or convex shape depending on initial panel perturbation (Figure 4.2). Panel equilibrium configurations at multiple values of $\lambda$ are shown in Figure 4.3. For $\lambda > 700$ the panel always converges to concave shape, suggesting that the upper branch with positive panel displacement in Figure 7 ceases to exist. Additionally, while the concave shape remains symmetric about the midpoint, the convex shape loses this symmetry with higher values of $\lambda$. This can be explained as follows.

For concave shaped deflections, the flow shows a high pressure region at the center of the panel due to flow deceleration (Figure 4.4(a)), resulting in a symmetric panel deflection profile. For the convex shape panel deflections, the flow accelerates over the plane, and at sufficiently higher $\lambda$ ($\sim 300$) a shock is formed slightly aft of the mid-point (Figure 4.4(b)) resulting in asymmetric panel deflection. With increasing $\lambda$ the shock moves to-
Figure 4.3

Panel deflection profiles for various dynamic pressures at $M = 0.9$.

Towards the trailing edge of the panel breaking the symmetry of flow over the panel, resulting in asymmetry of the panel deflection.

For $\lambda \leq 300$ the panel deflections are in good agreement with the divergence modes identified in [1]. For higher values of $\lambda$ the nonlinearity in the flow field (due to formation of shock) results in asymmetry of the convex shape. It is noted that the divergence mode reported in [1] is based on a linearized stability analysis to compute the bifurcation point at zero panel deflection where the nonlinear effects are absent.

4.1.2 Semi-Infinite Panel at $M = 1.2$

The response of the panel at Mach 1.2 is studied for increasing values of dynamic pressure, $\lambda$. Using the approach outlined above the plot of panel 3/4-chord displacement versus $\lambda$ in Figure 4.5 shows that the bifurcation point is approximately $\lambda = 16.25$. The branch of panel displacement below this value of $\lambda$ includes stable fixed points. As an
example, the response to an initial perturbation at $\lambda = 15$ in Figure 4.6 damps to zero displacement. Above the bifurcation point the response to a small perturbation causes the panel to settle into a LCO, indicating that the zero-displacement branch is unstable for $\lambda > 16.25$. The LCO amplitude increases with $\lambda$ while the nonlinear stiffening due to von Karman strain prevents its unbounded growth over time. The results are in close agreement with [16].

The panel displacement and nondimensional pressure in the flow field are plotted in Figure 4.7 for multiple instances during one flutter cycle at $\lambda = 17$. The pressure, which corresponds to the displacement of the panel at $\Phi = 0^\circ$, shows an expansion shock in the first half of the panel followed by a compression shock. Both the displacement and pressure contours are in good agreement with the flutter mode reported in [1], which was computed using a linearized bifurcation analysis. The time history and phase plot of the response at $\lambda = 30$ and 100 are shown in Figure 4.8 and Figure 4.9, respectively. The amplitude of the

Figure 4.4

Nondimensional pressure distribution for (a) concave and (b) convex deflections for $M = 0.9$ at $\lambda = 700$. 

(a)  
(b)
Nondimensional 3/4-chord panel displacement versus dynamic pressure at $M = 1.2$ shows the bifurcation point to be approximately $\lambda = 16.25$.

Time history of 3/4-chord displacement for $\lambda = 15$ shows a stable fixed-point.
panel response at the higher value of $\lambda = 100$ increases rapidly and settles into an LCO by about 1.5s, whereas it takes over 4s for the panel to settle into a LCO at $\lambda = 30$.

![Graph](image1)

**Figure 4.7**

(a) Panel displacement at three phases during one period of the flutter mode at $M = 1.2$ and $\lambda = 17$. (b) Nondimensional pressure in the flow field around the panel for $\Phi = 0^\circ$.

A frequency response of the time history of panel response at increasing value of dynamic pressure is used to compute the dominant frequencies. The frequency response in Figure 4.10(a) shows that dominant frequency remains nearly constant in the vicinity of the bifurcation point (close to $\lambda = 16.25$). This dominant frequency increases near linearly with higher values of $\lambda$ (Figure 4.10(b)) with stronger contribution coming from multiple higher frequencies. At the bifurcation point the dominant frequency is shown to be about $35Hz$, which is in close agreement with the nondimensional frequency reported in [1].

### 4.1.3 Semi-Infinite Panel at $M = 1.414$
Figure 4.8
(a) Time history of 3/4-chord displacement for $\lambda = 30, M = 1.2$ and (b) phase plot of the response.

Figure 4.9
(a) Time history of 3/4-chord displacement for $\lambda = 100, M = 1.2$ and (b) phase plot of the response.
At Mach 1.414 the bracketing procedure yields the bifurcation point at \( \lambda = 125 \). The flutter mode shape at this value of nondimensional dynamic pressure is plotted at four instants during one period in Figure 4.11(a). This mode is in agreement with those reported by Bhatia and Beran [1], who identified this flutter mode to be a single degree of freedom standing wave. The nondimensional pressure contour in the flow field around the panel is shown in Figure 4.11(b) for a displacement corresponding to \( \Phi = 0^\circ \). The sequence of compression and expansion shocks above the panel correspond to the positive and negative slope of the panel displacement. Increasing the dynamic pressure to \( \lambda = 560 \) causes the initial transient to be highly irregular (Figure 4.12) after which the panel settles into a LCO. The frequency response analysis of the panel time history in Figure 4.13 shows that the flutter mode at the bifurcation point of \( \lambda = 125 \) is dominated by a single frequency, which is consistent with the results in [1]. Bhatia and Beran demonstrated that the coupled system for \( 1.2 \leq M \leq 1.6 \) includes several hump modes where the flutter mode is composed of a single degree of freedom standing wave. This is more clearly represented by the higher
dominant frequency at $\lambda = 440$ in Figure 4.13(b). For higher value of $\lambda$ the impact of nonlinearity becomes stronger on the panel response with greater participation coming from multiple higher frequencies.

![Figure 4.11](image)

(a) Panel displacement at three phases during one period of the flutter mode at $M = 1.414$ and $\lambda = 125$. (b) Nondimensional pressure in the flow field around the panel for $\Phi = 0^\circ$.

### 4.1.4 Semi-Infinite Panel at $M = 1.6$

Consistent with the findings of Bhatia and Beran [1] the response at $M = 1.6$ is observed to be composed of a series of single degree-of-freedom hump-modes. The bracketing approach revealed the first bifurcation point to be approximately $\lambda = 210$ with a dominant contribution coming from the fifth structural mode, consistent with the finding of Bhatia and Beran [1]. At a higher dynamic pressure of $\lambda = 250$ the flutter mode was dominated by the sixth structural mode. This mode is plotted for different phase angles in a flutter cycle in Figure 4.14(a) along with the normalized pressure contours around the
Figure 4.12
(a) Time history of 3/4-chord displacement for $\lambda = 560, M = 1.414$ and (b) phase plot of the response.

Figure 4.13
$M = 1.414$ FFT magnitude of the panel response for (a) $75 \leq \lambda \leq 125$ and (b) $440 \leq \lambda \leq 560$. Dotted line for $\lambda = 125$ corresponds to the bifurcation point.
panel at $\Phi = 0^\circ$. At higher values of $\lambda > 800$ the panel response degenerates from a LCO to chaotic response (Figure 4.15). A frequency analysis of the time history at multiple values of $\lambda$ shows (Figure 4.16) the change in dominant frequencies at $\lambda = 210, 250$ and 640, which are representative of the mode switching due to the presence of multiple single degree-of-freedom flutter hump modes. For $\lambda > 800$ (Figure 4.16(b)) the contribution from a broad range of frequencies is characteristic of a chaotic response.

4.1.5 Semi-Infinite Panel at $M = 2.0$

The bracketing approach reveals the bifurcation point for $M = 2.0$ to be $\lambda = 540$. The flutter mode is plotted for multiple instants during the flutter cycle in Figure 4.17(a) and is consistent with the data in [1]. The normalized pressure contour around the panel at $\Phi = 0^\circ$ in Figure 4.17(b) shows the compression and expansion shocks convecting along
Figure 4.15
(a) Time history of 3/4-chord displacement for $\lambda = 880$, $M = 1.6$ and (b) phase plot of the response.

Figure 4.16
$M = 1.6$ FFT magnitude of the panel response for (a) $185 \leq \lambda \leq 250$ and (b) $640 \leq \lambda \leq 880$. Dotted line for $\lambda = 250$ corresponds to the bifurcation point.
the characteristic lines. A frequency response is used to identify the oscillation frequency and results are plotted for both Mach 1.8 and 2.0 in Figure 4.18.

![Figure 4.17](image)

(a) Panel displacement at three phases during one period of the flutter mode at $M = 2.0$ and $\lambda = 540$. (b) nondimensional pressure in the flow field around the panel for $\Phi = 0^\circ$.

### 4.1.6 Impact of Mach Number on Nondimensional Dynamic Pressure and Frequency for Bifurcation

The nondimensional dynamic pressure and frequency are compiled for a range of Mach numbers between 0.4 and 2.0, and compared with results from Bhatia and Beran [1]. The dynamic pressure in Figure 4.19 and frequency in Figure 4.20 show very good agreement with [1] across this range of flow conditions from subsonic to supersonic. For $M < 1$ the divergence instability occurs at zero frequency, which is not included in the frequency plot. The lowest instability dynamic pressure is observed at $M = 1$ with several changes in flutter modes between $M = 1$ and $M = 1.8$. At $M = 1.6$ the switch in flutter modes at $\lambda = 210, 260$ is shown with the filled and clear boxes.
Figure 4.18
FFT frequency variation for (a) $M = 1.8$ and (b) $M = 2.0$. Dotted line designates first indication of instability.

Figure 4.19
(a) Nondimensional dynamic pressure of bifurcation point of a semi-infinite panel for a range of Mach numbers and (b) a zoomed-in view around $M = 1$. 

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4.2 Shock Impingement

The response of a semi-infinite panel is studied at $M = 2$ under the impact of an impinging shock. Different angles of impinging shock are considered, which result in different values of pressure ratio, $P_3/P_1$, across the reflected shock. The results for inviscid flow with a pressure ratios of $P_3/P_1 = 1.2, 1.4,$ and $1.8$ are discussed in Section 4.2.1-4.2.3, respectively. The results for viscous flow with a laminar boundary layer and $P_3/P_1 = 1.4$ are discussed in Section 4.2.4 and the impact of boundary layer thickness is also studied.

**4.2.1 $M = 2.0$ Inviscid Flow with Oblique Shock Strength of $P_3/P_1 = 1.2$**

Figure 4.21 shows the converged solutions of static panel deformation in $M = 2.0$ inviscid flow, shock strength of $P_3/P_1 = 1.2$, and two dynamic pressures, $\lambda = 200$ and $875$; compared with the results in [32]. With the cavity pressure set to the average of the
zone 1 and 3 pressures, the panel is forced outward in a convex shape on the leading edge and forced inward in a concave shape on the trailing edge. The differential between the cavity pressure below the plate and the fluid pressures above the plate before and after the shock leads to the characteristic shape shown in Figure 4.21.

![Figure 4.21](image_url)

**Figure 4.21**

Static panel deformation for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.2$ for $\lambda = 200$ and $\lambda = 875$.

Figure 4.23 shows the pressure ratio ($P/P_1$) along the surface of the panel for both the dynamic pressures. The pressure ratio near the leading and trailing edges of the panel exhibit significant similarities with [32], while the near vertical jump in the pressure ratio near $X/L = 0.5$ indicates a much sharper capturing of the oblique shock at the impingement location than in [32]. The volume solution of pressure ratio ($P/P_1$) and Mach number for $\lambda = 875$ is shown in Figure 4.24. Predictions for $\lambda = 200$ and 875 (along with intermediate $\lambda$ values) for increasing shock strength are presented in the following sections, with Figure 4.25 containing the amplitude and Strouhal numbers for fluttering solutions. Since
neither prediction presented in this section result in flutter, the amplitudes are represented by a data point along the x-axis.

![Graph](image1)

![Graph](image2)

(a)  

(b)

**Figure 4.22**

Time history of panel deformation at 3/4-chord length for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.2$ for (a) $\lambda = 200$ and (b) $\lambda = 875$.

### 4.2.2 $M = 2.0$ Inviscid Flow with Oblique Shock Strength of $P_3/P_1 = 1.4$

Compared with the validation case of Subsection 4.2.2, which presents $\lambda = 875$ as a steady-state equilibrium solution for a weak shock ($P_3/P_1 = 1.2$), increasing the shock strength to $P_3/P_1 = 1.4$ attenuates the onset of flutter, where the resulting time history solution of the same $\lambda$ is given in Figure 4.26. Figure 4.27 shows the instantaneous deformation and surface pressures associated with two phases of the oscillation. $\Phi = 0^\circ$ and $\Phi = 180^\circ$ are compared with that of [20], which exhibits strong similarities in instantaneous mode shape and pressure distribution. The pressure contour for $\Phi = 180^\circ$ is also presented in Figure 4.28. A strong shock has formed at the panel leading edge, followed by
Surface pressure ratio \( (P/P_1) \) for \( M = 2.0 \) inviscid flow over a flexible panel subjected to an oblique impinging shock of strength \( P_3/P_1 = 1.2 \) and (a) \( \lambda = 200 \) and (b) \( \lambda = 875 \).

(a) Pressure ratio \( (P/P_1) \) and (b) Mach contours for \( M = 2.0 \) inviscid flow over a flexible panel subjected to an oblique impinging shock of strength \( P_3/P_1 = 1.2 \) and \( \lambda = 875 \). Vertical scale magnified 5x for clarity.
4.2.3 \( M = 2.0 \) Inviscid Flow with Oblique Shock Strength of \( P_3/P_1 = 1.8 \)

Figure 4.25
(a) Flutter amplitude and (b) Strouhal number predictions for \( M = 2 \) flow with \( P_3/P_1 = 1.2, 1.4, \) and 1.8

an equally strong recompression/expansion shock at the trailing edge. Flutter amplitudes and Strouhal numbers for the range \( \lambda = 100 - 875 \) can be found in Figure 4.25. There is a strong agreement with the benchmark, which has also been observed for instantaneous panel deformations and surface pressures for \( \lambda = 875 \).

Figure 4.29 shows the instability arising from strong shock resulting from an increase in the pressure ratio to \( P_3/P_1 = 1.8 \) for \( \lambda = 875 \). Figure 4.30 shows the panel deformation and the pressure ratio \((P/P_1)\) at two phase angles of an oscillation of the plate. The latter portions of the time history report near perfect periodicity which can be used to study the amplitude of the oscillations. These solutions agree qualitatively with [32] and [20] in which properties of the LCOs arising from a pressure ratio of 1.4 were presented. For \( \lambda = 875 \), the expectation for an increase in shock strength is a more drastic jump in surface
Figure 4.26

Time history of the panel deformation at 3/4-chord length for Mach 2.0 inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.4$ for $\lambda = 875$.

Figure 4.27

(a) Instantaneous panel deformation and (b) surface pressure ratio for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.4$ and $\lambda = 875$. 

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Figure 4.28

Pressure ratio contour for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.4$ and $\lambda = 875$. Vertical scale magnified 5x for clarity.

pressures and a corresponding increase in all panel deformation amplitudes with similar modal activity, which has been accurately captured. A snapshot of the pressure contour at $\Phi = 180^\circ$ is given in Figure 4.31.

A full range of dynamic pressures were simulated, in a manner similar to the uniform flow panel flutter case, in order to examine the bifurcation point of the system as well as the limit-cycle amplitude solutions. Figure 4.25 shows the bifurcation data comparison with that in [32]. The Loci-Chem/MAST analysis predicted higher amplitude than that reported in [32] over the range of dynamic pressures. However, the prediction of the critical $\lambda$ value matched that of [32]. The slight discrepancy in the amplitude may be due to a more accurate prediction of the mid-panel pressure discontinuity.

4.2.4 $M = 2.0$ Laminar Flow with Oblique Shock Strength of $P_3/P_1 = 1.4$
Figure 4.29

Time history of panel deformation at 3/4th chord length for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.8$ and $\lambda = 875$.

Figure 4.30

(a) Instantaneous panel deformation and (b) surface pressure ratio for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique impinging shock of strength $P_3/P_1 = 1.8$ and $\lambda = 875$ at phase angle of $\Phi = 0^\circ$ and $180^\circ$. 
Pressure ratio for $M = 2.0$ inviscid flow over a flexible panel subjected to an oblique shock of strength $P_3/P_1 = 1.8$ and $\lambda = 875$ at phase angle $\Phi = 180^\circ$. Vertical scale magnified 5x for clarity.

In the two cases studied in this section, the rigid panel solutions produced a boundary layer thickness of $\Delta_{LE} = 0.0156L$ and $\Delta_{LE} = 0.0262L$ at the leading edge of the panel in accordance with the setup in [20]. For $\Delta_{LE} = 0.0156L$, a viscous wall boundary condition was specified on the panel and the other co-planar boundaries in the fore and aft region of the panel so that the boundary layer naturally develops from the domain inlet. As a way to modulate a thicker boundary layer with the same fluid grid, the inlet boundary was initialized with a Blasius velocity distribution such that the desired boundary layer thickness was obtained at the panel leading edge. The rigid panel solutions for each of the boundary layer thicknesses are shown in Figure 4.32 and are consistent with that of [20]. In both the cases, separation vortices appear near the mid-panel, with the thicker boundary layer producing a much larger separation bubble.

Figure 4.33 shows the time histories of the panel displacement at 3/4-chord of the flexible panel for each of the boundary layer thicknesses. For $\Delta_{LE} = 0.0156L$, the snapshots
Figure 4.32

Nondimensional velocity distribution $u/u_1$ on a rigid panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$ and an oblique shock of strength $P_3/P_1 = 1.4$ for (a) $\Delta_{LE} = 0.0156L$ and (b) $\Delta_{LE} = 0.0262L$.

Figure 4.33

Time history of displacement at 3/4th chord of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$ and an oblique shock of strength $P_3/P_1 = 1.4$ for (a) $\Delta_{LE} = 0.0156L$ and (b) $\Delta_{LE} = 0.0262L$. 

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Figure 4.34
(a) Panel deformation and (b) nondimensional surface pressure distribution at $\Phi = 0^\circ$ (solid lines) and $\Phi = 180^\circ$ (dashed lines) of a LCO of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$ and an oblique shock of strength $P_3/P_1 = 1.4$ for $\Delta_{LE} = 0.0156L$. Solutions are compared with Li et al. [20].

of the panel deflection and surface pressure distribution at four phase angles of the LCO are shown in Figure 4.34. The corresponding nondimensional velocity magnitude, $u/u_1$, and temperature distribution in the fluid domain is shown in Figure 4.35. The plots show that at the beginning of an oscillation ($\Phi = 0^\circ$), the separation vortex is located near the panel midpoint. As the panel begins to deform, the separation vortex shifts toward the trailing edge and becomes compressed at $\Phi = 180^\circ$. The temperature distribution shows that the heating in the boundary layer and along the panel is nearly uniform with only a slight increase in temperature near the shock and trailing edge where the flow is undergoing compression. For $\Delta_{LE} = 0.0262L$, similar analysis was performed and is shown in Figure 4.37 and Figure 4.38. In this case, the separation bubble appears at the top of the fully convex panel at $\Phi = 0^\circ$. As the panel deforms, the bubble shifts towards the leading edge of the panel until around $\Phi = 180^\circ$ after which the bubble shifts towards the trailing edge of the panel.
Figure 4.35

Nondimensional velocity ($u/u_1$) distribution in the fluid domain at (a) $\Phi = 0^\circ$ (b) $\Phi = 90^\circ$ (c) $\Phi = 180^\circ$ and (d) $\Phi = 270^\circ$ of a LCO of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$, and an oblique shock of strength $P_3/P_1 = 1.4$ for $\Delta_{LE} = 0.0156L$. 
Figure 4.36

Temperature distribution in the fluid domain at (a) $\Phi = 0^\circ$ (b) $\Phi = 90^\circ$ (c) $\Phi = 180^\circ$ and (d) $\Phi = 270^\circ$ of a LCO of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$, and an oblique shock of strength $P_3/P_1 = 1.4$ for $\Delta_{LE} = 0.0156L$.

Figure 4.37

(a) Panel deformation and (b) nondimensional surface pressure distribution at various phase angles of a LCO of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$ and an oblique shock of strength $P_3/P_1 = 1.4$ with $\Delta_{LE} = 0.0262L$. Dotted lines represent comparison with Li et al. [20].
Figure 4.38

Nondimensional velocity \( (u/u_1) \) distribution in the fluid domain at (a) \( \Phi = 0^\circ \) (b) \( \Phi = 60^\circ \) (c) \( \Phi = 120^\circ \), (d) \( \Phi = 180^\circ \), (e) \( \Phi = 240^\circ \), and (f) \( \Phi = 300^\circ \) of a LCO of the flexible panel subjected to laminar flow at \( M = 2.0, \lambda = 875 \), and an oblique shock of strength \( P_3/P_1 = 1.4 \) for \( \Delta_{LE} = 0.0262L \).
Figure 4.39

Frequency spectrum of LCO for (a) $\Delta_{LE} = 0.0156L$ and (b) $\Delta_{LE} = 0.0262L$ for the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$, and an oblique shock of strength $P_3/P_1 = 1.4$.

Figure 4.40

Effect of boundary layer thickness on the Strouhal number of the LCO of the flexible panel subjected to laminar flow at $M = 2.0$, $\lambda = 875$, and an oblique shock of strength $P_3/P_1 = 1.4$. Clear data points represent secondary frequencies.
The frequency spectrum of the LCO is shown in Figure 4.39. It shows that the dominant frequency is 293.32 Hz for a leading edge boundary thickness of $\Delta_{LE} = 0.0156L$ along with a secondary frequency of 579.99 Hz. For $\Delta_{LE} = 0.0262L$, the LCO contains multiple secondary frequencies. The Strouhal number for the two cases is compared with that of [20] for both the $\Delta_{LE}$ as shown in Figure 4.40. Both cases predicted the Strouhal number corresponding to the dominant frequency in close agreement with the data in [20], with the exception of the secondary frequency for $\Delta_{LE} = 0.0156L$ which was appears to be missing from the analysis in [20]. Our analysis also predicts slightly lower values for the secondary frequencies in the $\Delta_{LE} = 0.0262L$ case.
CHAPTER 5
COMPARISON OF INVISCID, LAMINAR, AND TURBULENT BOUNDARY LAYER SOLUTIONS FOR FLUTTER PREDICTIONS

5.1 Solution Methodology

This section details the differences in laminar and turbulent boundary layer numerical predictions as it pertains to bifurcation point analysis and flutter for low Reynolds number flows. In a manner consistent with the previous validation studies, relevant similarities and differences in flutter development for each flow type have been analyzed. For previous supersonic Mach numbers, decreasing the dynamic pressure below the bifurcation point would lead to a stable, steady-state solution, whereas increasing the dynamic pressure above the bifurcation point would result in a limit-cycle (or time periodic) instability. The same point of this transition is of importance in this expansion of the previous results along with how the bifurcation point changes with flow type.

All cases in the following section reflect an impinging oblique shockwave of a medium strength shock, $P_3/P_1 = 1.4$, for $M = 2.0$ flow with viscous solutions considering $Re = 120,000$ and a boundary layer thickness of $\Delta_{BL} = 0.0156L$. Each data set contains solutions within the range $\lambda = 100 – 800$ in increments of 100 as well as $\lambda = 875$ except where the point of transition has been found to occur. Increments of $\lambda = 25$ were
used to narrow the range down for increased precision in the bifurcation point. Inviscid numerical solutions have been published for \( \lambda = 875 \), along with a select few laminar boundary layer thicknesses, which was previously used in the study. Those solutions are still valid, but this section expands upon those to include more emphasis on the bifurcation point, transition location, and the change in behavior of the coupled system as unsteady flow characteristics are introduced. In addition to the \( x \) by \( \dot{x} \) phase plot of prior cases, several cases for a laminar boundary layer exhibit unexpected behavior of the time history which necessitates a clearer visualization of the phase solution. The phase plot was therefore expanded to a 3D visualization where the \( x \) by \( \dot{x} \) are compared over time. This method more clearly expresses the behavior of the transients and steady state solutions.

For other characterizations of the flow solutions and for differentiation among laminar and turbulent simulations, the nondimensional pressure \( (P/P_1) \), instantaneous panel deformation over four phases of the oscillation \( (\Phi = 0^\circ, 90^\circ, 180^\circ \text{ and } 270^\circ) \), and the coefficient of skin friction, which is especially important in predicting the horizontal location of flow separation along the panel, was plotted for each solution. The skin friction is calculated using the following equation, where \( \tau_w \) is the wall shear stress, and \( u \) is the incoming freestream velocity of zone 1.

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho u^2}
\]  

(5.1)

Locations along the panel length in which the skin friction coefficient are zero are those where \( \frac{\partial u}{\partial y} \) (and the wall shear stress, \( \tau_w \)) is zero. These locations indicate flow separation
and reattachment locations. This data quantifies the extent of the separation bubble and influence of the flow separation on the system’s dynamic response.

The following subsections first contain a detailed approach to maintaining consistency of the boundary layer thickness while freestream velocity changes with an increase or decrease in dynamic pressure, and a grid resolution study and its effect on the bifurcation point as it relates to shock capturing. Immediately following, a full analysis for the range $\lambda = 100 - 875$ for inviscid, turbulent, and laminar flow types is detailed.

5.1.1 Boundary Layer Approach

As in the validation study, boundary layer solutions are initialized by a no-slip boundary condition co-planar with the panel with a Reynolds number of $Re = 120,000$, which is consistent with [20]. Additionally, the boundary layer thickness for comparison is given as $\Delta_{BL} = 0.0156L$, which has been previously enforced by appropriately shortening the domain upstream of panel and allowing the boundary layer to naturally develop due to the no-slip boundary condition. URANS solutions utilized Menter’s SST turbulence model for turbulence closure (which is a well validated two-equation model useful for large pressure gradients and for flows which are prone to separation) and a constant viscosity transport model, where a specified laminar conductivity, $k_{cond}$ (from Equation (3.16)), and dynamic viscosity, $\mu$ (from Equation (3.19)), define the freestream Reynolds number.

The Reynolds number is enforced between changes in $\lambda$ by adjusting $k_{cond}$ and $\mu$ each time. For example, for a specified $\lambda$, the relevant flow parameters are calculated and are
then used to calculate the corresponding conductivity and dynamic viscosity to define a $Re = 120,000$ for each independent set of flow conditions.

A $y^+$ of 1 is typically recommended as a maximum for turbulence modeling. The first grid point of the domain is located at $y = [0, 10^{-5}]m$, and the following is a calculation for the wall normal spacing with the given grid and flow parameters. For a $Re = 120,000$ and a dynamic pressure of $\lambda = 875$, the coefficient of friction based on analytical solutions for flat-plate boundary layer theory is:

$$C_f = \frac{0.026}{Re_x^{1/7}} = 0.00489 \quad (5.2)$$

The analytical wall shear stress is therefore:

$$\tau_w = C_f \rho U_\infty^2 = \frac{0.00489 \times 0.54 \times 292.35^2}{2} = 112.861 \frac{N}{m^2} \quad (5.3)$$

The friction velocity is then determined as:

$$U_f = \sqrt{\frac{\tau_w}{\rho}} = 14.456 \frac{m}{s} \quad (5.4)$$

The wall normal spacing recommended as a maximum for this flow is given as:

$$\delta_s = \frac{y^+ \mu}{U_f \rho} \approx 5 \times 10^{-5}m \quad (5.5)$$

For a wall normal spacing of $10^{-5}$, as is used for the preceding and subsequent viscous simulations, the effective $y^+$ is 0.20, thereby ensuring that this grid is sufficient for unsteady flows in terms of wall normal spacing. One aspect not considered in present work
was whether the grid resolution near the panel was sufficient to capture the small-scale fluctuations of the turbulence. This is one potential source of error that might present itself in the unsteady solutions and might warrant a closer examination of the grid resolution in future work.

To ensure that the preceding process results in the desired boundary layer profiles, the laminar boundary layer at the panel leading edge was extracted and nondimensionalized for a range of dynamic pressures. Figure 5.1 shows the effective boundary layers nondimensionalized by their respective freestream velocities.

![Converged boundary layer profiles for $\lambda = 100$–800. Profiles have been nondimensionalized by respective freestream velocity, $u_1$.](image)

For $\lambda = 100$–800, the boundary layer profiles are nearly identical. Additionally, for a desired boundary layer thickness of $\Delta_{BL} = 0.0156L$, which equates dimensionally to $\Delta_{BL} = 0.00468m$ for a panel length of $0.3m$, 99% of the freestream reflects the proper boundary layer thickness.
A uniform flow, rigid panel solution for $M = 2.0$, $\lambda = 875$, and $Re = 120,000$ flow with and without turbulence modeling was analyzed to ensure the characteristics of turbulence have been appropriately resolved with the inclusion of the $k-\omega$ SST model. Figure 5.2 shows the difference in boundary layer profiles at the panel leading edge. This trend indicates that for turbulence modeling, the boundary layer profile at the leading edge was approximately the same as that of the laminar solution, and the boundary layer thickness is additionally equivalent.

![Figure 5.2](image)

**Figure 5.2**

Laminar and turbulent boundary layer profiles at panel leading edge for $M = 2.0$, $\lambda = 875$ and $Re = 120,000$.

To characterize the difference in the development of the boundary layer across the panel surface, the velocity profile data at both the leading and trailing edge were compared (seen in Figure 5.3) and nondimensionalized by their respective boundary layer thickness. At the leading edge, as was observed in Figure 5.2, the boundary layer profiles are nearly identical. At the trailing edge, the laminar solution maintains a nearly identical velocity...
profile to that of the leading edge, whereas for turbulence modeling, the alterations in the profile indeed show characteristics consistent with the development of a turbulent boundary layer, including a significant increase in boundary layer thickness and increase in near-wall velocity gradient. Figure 5.4 shows the activation of the turbulent eddy viscosity across the panel surface, where the viscous effects are clearly decreasing the momentum of the fluid. Both observations indicate the turbulence modeling is behaving as expected.

![Figure 5.3](image.png)

Figure 5.3

Development of laminar and turbulent boundary layer profiles at panel leading and trailing edge for $M = 2.0$, $\lambda = 875$ and $Re = 120,000$. Vertical scaling has been nondimensionalized by respective boundary layer thickness.

### 5.1.2 Grid Resolution Study

A full validation study was performed for $P_3/P_1 = 1.4$ for a range of $\lambda = 100–875$ for inviscid flows. Also included is the effect of grid resolution on the bifurcation point and shock resolution. The coarse, medium, fine, and finest grid resolutions are bound by the points extending from $x = [1.32, 3.3]m$ and $y = [0, 3.3]m$. For all domains, the first
Figure 5.4

Normalized kinetic eddy viscosity distribution for $M = 2.0$, $\lambda = 875$ and $Re = 120,000$.

grid point begins at $y = [0, 10^{-5}]m$. The coarsest grid used a stretched Cartesian mesh with a cell distribution of $326 \times 213$. Beginning with the medium grid type, the fully structured mesh was converted to a hybrid mesh by repositioning the Cartesian mesh portion to the near plate region (bound by the points extending from $x = [1.32, 1.98]m$ and $y = [0, 0.26]m$) such that the plate was centered inside the Cartesian mesh, and that the incoming and reflected shock would enter and exit the domain through the side boundaries. The remainder of the domain (except for a small portion set aside for the boundary layer near the wall) was populated with unstructured grid cells. The original coarse grid geometry was maintained to prevent unforeseen numerical artifacts from emerging when interacting with much closer domain boundaries. The structured portion of the domain uses a cell distribution of $341 \times 183$, where the total cell count including those outside the near plate region is approximately $80,000$ cells. The fine and finest grids used the same hybrid mesh structure and dimensioning, with the near wall region containing the
cell distribution $482 \times 259$ and $681 \times 365$ for fine and finest, respectively. The total cell count for the fine grid was 156,000 and 307,000 for the finest. Table 5.1 contains relevant information for each grid type, and Figures 5.5 and 5.6 indicate the domain coordinates and cell densities associated with each refinement level compared with the grid sizes of the benchmark numerical solutions.

Table 5.1: Detailed Grid Types

<table>
<thead>
<tr>
<th>Figure Reference</th>
<th>Coarse</th>
<th>Medium</th>
<th>Fine</th>
<th>Finest</th>
<th>Visbal [32]</th>
<th>Li [20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near Plate Size</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grid Size</td>
<td>60k</td>
<td>62k</td>
<td>120k</td>
<td>240k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Near Plate Coordinates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X/L</td>
<td></td>
<td>[-0.6 – 6]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y/L</td>
<td></td>
<td>[0 – 0.867]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain Coordinates</td>
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<td></td>
<td></td>
<td></td>
<td>[-0.5 – 1.5]</td>
<td></td>
</tr>
<tr>
<td>X/L</td>
<td></td>
<td>[-0.6 – 6]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y/L</td>
<td></td>
<td>[0 – 5]</td>
<td></td>
<td></td>
<td></td>
<td>[0 – 0.35]</td>
</tr>
<tr>
<td>Plate Coordinates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X/L</td>
<td></td>
<td>[0 – 1]</td>
<td></td>
<td></td>
<td></td>
<td>[0 – 1]</td>
</tr>
<tr>
<td>Y/L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spatial Order of Accuracy</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>3 (MUSCL)</td>
</tr>
<tr>
<td>$P_3/P_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.4</td>
</tr>
<tr>
<td>Regime</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Inviscid</td>
</tr>
</tbody>
</table>

The most distinct difference between subsequent grids is the resolution of the shock discontinuity at the impingement location on the panel surface. Dissipation of the shock can negatively affect the solution accuracy, so it is important to maintain as close a representation of the discontinuity as possible. To examine how well the shock is being resolved between cases, the steady-state, rigid panel solution is extracted, and the pressure ratio across the panel surface is plotted for each grid resolution. Figure 5.7 shows the convergence of the shock to the expected discontinuity in pressure. As the grid is refined, the shock approaches a step function-like discontinuity, as expected according to the zonal pressures specified in the domain.
Figure 5.5
Coarse grid with dimensional units ($m$).

(a) Full-size medium grid type with dimensional units ($m$). Near plate regions for (b) medium, (c) fine, and (d) finest grid types.
Figure 5.7

Converged rigid solution shows effect of grid refinement on shock capturing.

For comparison of each grid type in practice, a handful of solutions were compared with predictions of Visbal [32] and Li et al. [20]. According to Visbal [32], for a pressure ratio of 1.4 for inviscid flows, the stable/LCO branch is highly sensitive to initial conditions. Mainly, for the branch for $P_3/P_1 = 1.4$ to fall between 1.2 and 1.8, the simulation must initially be progressed to a steady-state, rigid panel solution. This solution is saved and used as the initial condition for simulation with a compliant panel. A range of solutions for various dynamic pressures were then produced with each grid type, and the bifurcation point, maximum and minimum y-displacements at $3/4$-chord, total amplitude, and Strouhal number were compared with published numerical solutions.

The bifurcation point for each grid type is given below, with the intermediate value between stable and LCO chosen as an approximate bifurcation point. For a more conclusive
bifurcation point analysis, the bracketing approach used previously for subsonic uniform validation cases should be employed here as well. Visbal [32] lists the critical $\lambda$ value as 575. The coarsest grid type has an approximate bifurcation point of 365, followed by the medium with 513, the fine with 538, and the finest with 560. Li et al. [20] lists the bifurcation point at 625. Table 5.2 contains the bifurcation bounds, approximate bifurcation points, and percent difference of MAST/Chem and Li et al. [20] numerical solutions compared with Visbal [32], along with an approximate computational time required for obtaining a solution for a single $\lambda$ for 800,000 time steps (or up to 2s) for laminar boundary layers. While inviscid and turbulent solutions do not require nearly as many time steps to achieve a converged solution, the approximate computational time considers the worst-case scenario. There is some convergence in bifurcation point for increasing refinements, however, an approximate solution with similar characteristics of the benchmark clearly outweighs the computational resources required to achieve the highest fidelity solution. To select an appropriate grid with a modest balance of solution accuracy and computational efficiency, the following set of solutions were compared for each grid.

Table 5.2: Bifurcation point percent difference for increasing grid resolution.

<table>
<thead>
<tr>
<th>Grid/Case</th>
<th>Last Stable</th>
<th>First LCO</th>
<th>Approx. Bif. Point</th>
<th>Approx. % Difference</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>350</td>
<td>365</td>
<td>357.5</td>
<td>37.80%</td>
<td>$\approx$ 61 hrs.</td>
</tr>
<tr>
<td>Medium</td>
<td>500</td>
<td>525</td>
<td>512.5</td>
<td>10.90%</td>
<td>$\approx$ 100 hrs.</td>
</tr>
<tr>
<td>Fine</td>
<td>525</td>
<td>550</td>
<td>537.5</td>
<td>6.50%</td>
<td>$\approx$ 197 hrs.</td>
</tr>
<tr>
<td>Finest</td>
<td>550</td>
<td>570</td>
<td>560</td>
<td>2.60%</td>
<td>$\approx$ 216 hrs.</td>
</tr>
<tr>
<td>Li et al. [20]</td>
<td>-</td>
<td>-</td>
<td>625</td>
<td>8.70%</td>
<td>-</td>
</tr>
</tbody>
</table>
For each grid type, the maximum and minimum y-displacement deflection of the 3/4-chord for a range of $\lambda$ is shown in Figure 5.8. The coarsest grid shows large deviations in the amplitude from subsequent refinements. As the grid is refined, the bifurcation point is asymptotically increasing closer to the critical value given by Visbal [32] of $\lambda = 575$. Past bifurcation, flutter solutions share similar maximum amplitudes among each refinement (except for coarse) and with Li et al. [20]. However, these solutions exhibit larger minimum deflection than predicted. This is further evidenced by a comparison of the instantaneous panel deformation and surface pressure ratio (seen previously in Figure 4.28) for $\lambda = 875$ using the medium grid type with data provided by Li et al. [20].

There are apparent differences in instantaneous panel deformation, most notably a slight longitudinal shift in the convex portion toward the leading edge for $\Phi = 0^\circ$, and a slight increase in the maximum displacement of the concave portion toward the trailing edge for $\Phi = 180^\circ$. There are also differences in the surface pressure distribution. MAST/Chem solutions show a large, sharper increase in pressure at the leading edge and impingement location with another sharp decrease at trailing edge compared to that of Li et al. [20].

In addition to these maximum and minimum deflections, the amplitude of the 3/4-chord deflections is given in Figure 5.10, which considers amplitudes compared with both Visbal [32] and Li et al. [20]. For the data provided, the discrepancy between maximum and minimum amplitudes with Li et al.[20] becomes more obvious. For each $\lambda$, the mean deflection is higher overall compared with both Visbal [32] and Li et al.[20], but there is a much closer matching between MAST/Chem and Visbal [32].
Figure 5.8

Maximum and minimum deflections at 3/4-chord for $M = 2.0$ and $P_3/P_1 = 1.4$ for all grid resolutions.

Figure 5.9

Mean amplitude for LCO cases compared with Visbal [32] and Li et al. [20] for $M = 2.0$ and $P_3/P_1 = 1.4$ for all grid resolutions.
The Strouhal number comparison (Figure 5.10) shows a comparable trend for increasing dynamic pressures. Each grid shows a decrease in $St$ for an increase in $\lambda$ which is consistent with Visbal [32] and Li et al. [20]. There is a distinct separation in the Strouhal number between coarse and the medium to finest grid resolutions, and interestingly a closer matching between coarse and the benchmark solutions. However, from the medium to finest grid, there is an obvious convergence in Strouhal number.

![Figure 5.10](image)

**Figure 5.10**

Strouhal number for increasing nondimensional dynamic pressure compared with Visbal [32] and Li et al. [20] for $M = 2.0$ and $P_3/P_1 = 1.4$ for all grid resolutions.

For further evidence of convergence, if we compare the magnitudes of the maximum and minimum 3/4-chord deflections in terms of a convergence ratio ($R$) for increasing refinement levels, there is a clear convergence in both cases as seen in Table 5.3, where a convergence ratio $0 < R < 1$ indicates monotonic convergence. The bracketing approach bounds as it is currently presented does not permit an accurate conclusion to be drawn from...
the bifurcation point data, and similarly there are limitations in the fast Fourier transform process that do not allow for a wider range of frequencies to be captured, so bifurcation point and Strouhal number cannot presently be used as a convergence parameter. Despite this, a convergence has been observed for this range of fluid grid resolutions indicating that we are within the asymptotic range of convergence, and further solutions may be trusted. Overall, the range of solutions examined in the grid refinement study show that for cases not near the bifurcation point, apart from the coarse grid, characteristics of the panel do not drastically change with increasing levels of refinement. Additionally, with a percent difference of around 10%, we can have confidence in the corresponding viscous solutions of the same dynamic pressure. Therefore, the medium grid type was found to contain an equal balance between computational cost and solution accuracy, so for comparison of inviscid, laminar, and turbulent solutions, the medium grid type is employed as a baseline.

<table>
<thead>
<tr>
<th></th>
<th>Coarse</th>
<th>Medium</th>
<th>Fine</th>
<th>Finest</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max.</td>
<td>-0.135</td>
<td>0.164</td>
<td>0.181</td>
<td>0.190</td>
<td>0.529</td>
</tr>
<tr>
<td>Min.</td>
<td>-2.023</td>
<td>-1.818</td>
<td>-1.783</td>
<td>-1.768</td>
<td>0.429</td>
</tr>
<tr>
<td>Avg.</td>
<td>-1.079</td>
<td>-0.827</td>
<td>-0.801</td>
<td>-0.789</td>
<td>0.462</td>
</tr>
</tbody>
</table>

5.2  \( M = 2.0 \) Inviscid flow with an Oblique Shock Strength of \( P_3/P_1 = 1.4 \)

For \( P_3/P_1 = 1.4 \), published numerical solutions have shown that the transition from steady-state equilibrium to a time periodic response is heavily dependent on the initial con-
ditions. The same has been found to be true in this data set. Like the previous validation cases, in order to obtain this particular response, the computational domain was progressed to a steady state with a panel which is initially rigid. The steady state solution was then used as the initial condition. Even though no boundary layer exists in the following inviscid solutions, this procedure was used to maintain consistency of solutions. The following subsections detail a full analysis of $\lambda = 100 - 875$. Section 4.2.4 contains the validation of the present FSI solver compared with published numerical predictions for this set of conditions including flutter amplitudes, Strouhal numbers, deformations, and surface pressures. The bifurcation point for this data set is approximately 512.5 which was found using the bracketing approach.

5.2.1 Steady-State Equilibrium Solutions

For the range $\lambda = 100 - 500$, the panel converges to a stable, steady-state equilibrium solution. Three instances of the solution ($\lambda = 100, 300, \text{ and } 500$) are plotted together for exhibit the development of various solutions as the dynamic pressure is increased up to the bifurcation point. Figure 5.11 shows the evolution of the equilibrium solution over time at the $3/4$-chord, with the $\delta/H$ amplitude asymptotically decreasing as the bifurcation point is reached (between $\lambda = 500$ and $\lambda = 525$). Similarly, the vertical panel deformation across the panel length is shown in Figure 5.12 and reveals the same trend. As observed in prior oblique shock cases, the differential between cavity pressure and the zonal pressures associated with the freestream forces the panel into a convex shape at the leading edge and
a concave shape at the trailing edge. The inflection point for the deformation is located slightly aft the mid-chord, which remains consistent with increases in dynamic pressure. The 2D and 3D phase plots are shown in Figure 5.13.

![Figure 5.11](image)

Figure 5.11

Time history of 3/4-chord displacement for $\lambda = 100$, 300, and 500 for inviscid flows.

For each case in this range, the resolution of the discontinuity across the midpoint is not dependent on the nondimensional dynamic pressure of the system. According to Figure 5.12, increasing the nondimensional dynamic pressure slightly increases the jump in pressure at the leading edge followed by a decrease in pressure at the trailing edge. Figure 5.14 shows the corresponding pressure contour of $\lambda = 100$ and is representative of other steady-state equilibrium solutions within the given range.

5.2.2 Time Periodic Solutions
Figure 5.12

(a) Panel deformation and (b) surface pressure ratio for $\lambda = 100, 300,$ and $500$ resulting from a steady-state equilibrium for inviscid flows.

Figure 5.13

(a) 2D and (b) 3D phase plot for $\lambda = 100, 300,$ and $500$ resulting from a steady-state equilibrium for inviscid flows.
For $\lambda \geq 525$, a limit-cycle instability forms, revealing a bifurcation point that exists between 500 and 525. Since $\lambda = 875$ was used in the validation study to compare with [20] for $P/P_1 = 1.2$ and 1.8, this $\lambda$ is used as the representative for all LCO cases with inviscid flow conditions. Figure 5.15 shows the time history of the $3/4$-chord. It is apparent from the periodic behavior in the LCO amplitude that there exists an influence of more than one frequency which presents itself in the form of a slightly lower secondary amplitude of oscillation. The minute difference in amplitude suggests these frequencies are closely tied and that instantaneous solutions of the two amplitudes will not show significant variations in comparison. The periodicity also remains consistent in the latter stages of the time history. Figure 5.16(a) shows the instantaneous panel deformation over four phases of the oscillation, where a single oscillation is considered as the larger amplitude deflection. The deformation of $\Phi = 0^\circ$ contains a large, non-symmetric, convex deflection, followed by a slight convex curvature at the trailing edge. $\Phi = 180^\circ$ becomes more symmetric.
with a convex leading edge and concave trailing edge that extends more than twice the
panel thickness in both directions. Both phases are similar to the limit-cycle instability
of \( P/P_1 = 1.8 \) in Figure 4.30. The phase plots (seen in Figure 5.17) show a closed path
trajectory consistent with a time periodic dynamic response.

![Figure 5.15](image)

**Figure 5.15**

Time history of 3/4-chord displacement for \( \lambda = 875 \) for inviscid flows.

The pressure ratio across the panel surface shows \( P/P_1 \) over four phases of the oscilla-
tion (Figure 5.16(a)), with Figure 5.18 showing the corresponding pressure contours in the
volume. Similarly to instantaneous deformation plot, the behavior of \( P/P_1 \) in Figure 5.16
for \( \Phi = 0^\circ \) and \( \Phi = 180^\circ \) is characteristically consistent with what was previously seen in
the LCO solution of \( P/P_1 = 1.8 \) in Figure 4.30.

5.3 \( M = 2.0 \), Low Re flow with an Oblique Shock Strength of \( P_3/P_1 = 1.4 \) and a
Developing Turbulent Boundary Layer
Figure 5.16
(a) Panel deformation and (b) surface pressure ratio for $\lambda = 875$ resulting from a dynamic instability for inviscid flows.

Figure 5.17
(a) 2D and (b) 3D phase plot for $\lambda = 875$ resulting from a dynamic instability for inviscid flows.
Figure 5.18

Nondimensional pressure distribution for $\lambda = 875$ resulting from a dynamic instability for inviscid flows where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.
To the author’s knowledge, the only published numerical solutions of this kind detail those for inviscid flows, as well as a handful of cases with a laminar boundary layer, but only for a single nondimensional dynamic pressure ($\lambda = 875$) and boundary layer thickness ($\delta_{BL} = 0.0156 * L$). Therefore, the following section details a range of nondimensional dynamic pressures for a laminar boundary layer with the inclusion of turbulence modeling. The methodology outline in Section 5.1.1 were followed to maintain consistency of the boundary layer at the panel leading edge, as well as to show the activation of turbulent kinetic eddy viscosity in the domain such that the boundary layer growth is consistent with expectations. The bifurcation point was again found to exist between 500 and 525 (approximately 512.5) as in the inviscid predictions.

5.3.1 Steady-state Equilibrium Solutions

The time history (shown in Figure 5.19) shows a steady-state equilibrium for all solutions up to $\lambda = 500$, the same as was determined for this range of $\lambda$ for inviscid predictions. Figure 5.20 also shows the same convergent behavior of the panel deformation, although with a slightly lower maximum deflection at the leading edge. This could be attributed to the formation of the shock as the boundary layer interacts with the panel’s edge. The 2D and 3D phase plots can be seen in Figure 5.21. For $\lambda = 500$, the transient behavior of the time history indicates that the bifurcation may be closer to $\lambda = 500$ for flows with a turbulent boundary layer as opposed to inviscid. A more rigorous bracketing approach would be required to fully investigate these characteristics.
Figure 5.19

Time history of 3/4-chord displacement for $\lambda = 100$, 300, and 500 for a turbulent boundary layer.

Figure 5.20

(a) Panel deformation and (b) surface pressure ratio for $\lambda = 100$, 300, and 500 resulting from a steady-state equilibrium for a turbulent boundary layer.
Figure 5.21

(a) 2D and (b) 3D phase plot for $\lambda = 100$, 300, and 500 resulting from a steady-state equilibrium for a turbulent boundary layer.

The associated surface pressure distribution and pressure contour in the volume are shown in Figure 5.20 and Figure 5.22, respectively. Again, Figure 5.14 shows the pressure contour of $\lambda = 100$ and is representative of other steady-state equilibrium solutions. Figure 5.20(b) shows significant similarity to that of the inviscid prediction for the same $\lambda$, although with a more attenuated jump and decrease at the leading and trailing edge, respectively. This is likely due to the influence of the boundary layer and increased mixing attributed to activation of the turbulence. This behavior is also present in the pressure contours, where there is small but noticeable diffusion of the incoming shock near the panel’s surface. The friction coefficient for $\lambda = 100$, 300, and 500 is shown in Figure 5.23 along the panel surface. Since the friction coefficient is positive ($C_f > 0$), there is no evidence of flow separation, and the influence of the boundary layer is minimal overall.
Figure 5.22

Nondimensional pressure distribution for $\lambda = 100$ resulting from a steady-state equilibrium for a turbulent boundary layer.

Figure 5.23

Friction coefficient for $\lambda = 100$, 300, and 500 resulting from a steady-state equilibrium for a turbulent boundary layer.
5.3.2 Time Periodic Solutions

For $\lambda \geq 525$, a limit-cycle instability forms, revealing a bifurcation point that exists between 500 and 525 much in the same way as that of inviscid predictions. $\lambda = 875$ is once again considered as the representative time periodic response for this flow type. Figure 5.25 shows near perfect periodicity in the time history response of the 3/4-chord, and any influence of a secondary flutter amplitude (as in the inviscid solution of the same $\lambda$) has been diminished. In comparison with the inviscid solution, (see Figure 5.15) the maximum and minimum deflections of the 3/4-chord is much higher with the turbulent boundary layer. Figure 5.26 shows the instantaneous deformation over four phases of the oscillation. $\Phi = 0^\circ$ shows a nearly symmetric convex deflection at both the leading and trailing edge, with the impingement location ($X/L = 0.5$) shifted in the -y direction. $\Phi = 180^\circ$ is more characteristically similar in shape in comparison with the inviscid, although with a much larger concave deformation at the trailing edge that extends 3 times
the panel thickness. The phase plots for this response can be seen in Figure 5.27 where the perfect periodicity attributed to a closed path trajectory can be more easily identified.

![Figure 5.25](image)

**Figure 5.25**

Time history of 3/4-chord displacement for $\lambda = 875$ for a turbulent boundary layer.

The surface pressures found in (Figure 5.26(b)) reveals similar attenuation of the shock through each oscillation due to the addition of the turbulence modeling as compared with steady-state equilibrium solutions. Figure 5.28 reveals the corresponding pressure contours in the volume, for which the small diffusion of the shock near the panel’s surface is present again. The friction coefficient (Figure 5.29) indicates that for all $\Phi$, $C_f > 0$ which again shows no signs of flow separation. With the dip in friction coefficient near the panel mid-chord, it seems that the $P_3/P_1$ is not sufficiently strong to induce separation in any case with a developing turbulent boundary layer.
Figure 5.26
(a) Instantaneous panel deformation and (b) surface pressure ratio for $\lambda = 875$ resulting from a dynamic instability for a turbulent boundary layer.

Figure 5.27
(a) 2D and (b) 3D phase plot for $\lambda = 875$ resulting from a dynamic instability for a turbulent boundary layer.
Figure 5.28

Nondimensional pressure distribution for $\lambda = 875$ resulting from a dynamic instability for a turbulent boundary layer where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.

Figure 5.29

Friction coefficient for several phase angles resulting from a dynamic instability for a turbulent boundary layer.
Figure 5.30

Nondimensional velocity distribution for $\lambda = 875$ resulting from a dynamic instability for a turbulent boundary layer where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.
5.4 $M = 2.0$, Low $Re$ flow with an Oblique Shock Strength of $P_3/P_1 = 1.4$ and a Laminar Boundary Layer

Numerical predictions for this set of conditions already exist in literature for $\lambda = 875$ and were previously detailed in the validation study of Section 4.2.4, but the ultimate goal of this section is to quantify how slight changes in the nondimensional dynamic pressure affect the flutter boundaries due to the inclusion of a purely laminar boundary layer. Numerical solutions for $\lambda = 875$, as well as those found in [20], show a well-defined time periodic response. Based on observations of the inviscid and turbulent solutions, one would assume that the laminar boundary layer would also exhibit a clear transition from steady-state equilibrium to time periodic, but this has not been observed. In fact, only two predictions ($\lambda = 100$ and $\lambda = 875$) exhibit the traditional steady-state equilibrium and time periodic response. Intermediate solutions instead reveal influences of multiple modes and frequencies which ultimately contaminate the transient solution and induce chaotic flow from an otherwise steady-state equilibrium or fluttering solution. Initially observed data showed that after 1 second, some predictions exhibited typical behavior of a steady-state equilibrium and LCO. However, as more cases were observed and it became apparent that no clear bifurcation point exists, each solution was progressed up to 2 seconds to allow ample time for full development of the solution. In doing so, it became clear that although certain solutions initially appeared to be a steady-state equilibrium or LCO, the system ultimately transitioned to chaotic flow. The following subsections detail solutions containing traditional steady-state equilibrium and LCO solutions as well as the solution
branches for chaotic flow. The solutions have been divided into categories which reflect similar development of transient and steady-state features of the 3/4-chord time history.

5.4.1 Feature 1: Steady-state Equilibrium Solution

The time history for $\lambda = 100$ (Figure 5.31) shows convergence of the time history to a steady-state equilibrium response. The converged $\delta/H$ is very similar to that of the developing turbulent boundary layer for $\lambda = 100$ in Figure 5.19. Figure 5.32 shows the converged static panel deformation of the steady-state equilibrium response, which contains the same dominant mode as that of inviscid or turbulent predictions. Like previous sets of solutions, the phase plots can be referenced in Figure 5.33 to show the steady-state convergence of the 3/4-chord.

Figure 5.31
Time history of 3/4-chord displacement for $\lambda = 100$ for a laminar boundary layer.
Figure 5.32
(a) Panel deformation and (b) surface pressure ratio for $\lambda = 100$ resulting from a steady-state equilibrium for a laminar boundary layer.

Figure 5.33
(a) 2D and (b) 3D phase plot for $\lambda = 100$ resulting from a steady-state equilibrium for a laminar boundary layer.
$P/P_t$ (Figure 5.32(b)) and the corresponding pressure contour (Figure 5.34) show an interesting characteristic of the laminar solutions which differentiates this set of solutions apart from other cases of the same dynamic pressure. For the converged solution, the jump and sharp decrease in pressure at the leading and trailing edge, respectively, have been nearly eliminated, and intermediate values of pressure begin approach a linear behavior. This is indicative of some significant diffusion of the shock near the plate, and that the shock has not been entirely maintained in the latter stages of the solution. One explanation might lie in the findings of Figure 5.35, where the coefficient of friction indicates a large separation that spans from $0.3 < X/L < 0.8$, or around 50% of the plate length.

![Figure 5.34](image)

Nondimensional pressure distribution for $\lambda = 100$ resulting from a steady-state equilibrium for a laminar boundary layer.

### 5.4.2 Feature 2: Steady State Equilibrium to LCO to Chaos
Figure 5.35
Friction coefficient for \( \lambda = 100 \) resulting from a steady-state equilibrium for a laminar boundary layer.

Figure 5.36
Nondimensional velocity contour for \( \lambda = 100 \) resulting from a steady-state equilibrium for a laminar boundary layer.
This branch of solutions encompasses those in the range $\lambda = 200 - 400$, where the evolution of the time history appears to begin with a converging steady-state solution, which diverges first to an LCO and ultimately to chaos. Figure 5.37 contains the time history solutions of each $\lambda$ in this range and the associated frequency spectra. $\lambda = 400$ is considered as an example representation of the solutions in this range.

For $\lambda = 400$, a sample of the chaotic time history is shown in Figure 5.38(a) with several sample locations (S1-S4) for which the instantaneous panel deformation and surface pressure ratio are shown for the respective sample location. Although chaotic flow is traditionally unorganized and unpredictable, there is some organization to the oscillations of the deformation and surface pressures, where low amplitude, high frequency oscillations have taken shape, and the panel deforms about some mean panel deflection. Further characterization of this observed behavior can be found in the following section (Section 5.4.3) where this behavior has also been shown to occur.

5.4.3 Feature 3: Primary LCO to Secondary LCO to Chaos

This branch of solutions details the range $\lambda = 500 - 800$ which is characterized by an initial, primary LCO response which transitions to some secondary LCO for a short period of time before ultimately diverging to a chaotic flow. Figure 5.39 contains the time history and frequency spectra solutions for each $\lambda$ in this range, and $\lambda = 800$ is considered as the representative case for further examination. Since both features 2 and 3 result in a chaotic flow with the only difference between the two being the transient solutions and how the
Figure 5.37

Time history response and frequency spectra of (a-b) $\lambda = 200$, (c-d) $\lambda = 300$, and (d-e) $\lambda = 400$ for a laminar boundary layer.
Figure 5.38
(a) Time history slice of chaotic response with arbitrary sample locations for instantaneous (b) panel deformation and (c) surface pressure ratios for $\lambda = 400$ for a laminar boundary layer.
resulting chaos has developed in time, graphical representations of various flow solutions (i.e. $P/P_1$, $C_f$, and $u/u_1$) can be considered to be representative of both Sections 5.4.2 and 5.4.3. Evidence of this claim is found in the similarities of the instantaneous panel deformation and surface pressure ratio of Figure 5.40 compared with that of Figure 5.38.

Observation of Figure 5.39(g) and Figure 5.39(h) for $\lambda = 800$ reveals a dominant primary frequency likely that of the primary mode, in addition to several high frequency influences. Figure 5.40(b) contains a snapshot of the panel deformation at several sample locations (S1-S4) as designated by Figure 5.40(a), where the influence of the high frequency oscillations becomes immediately apparent as several low amplitude modes have appeared across the panel surface. This instantaneous panel deformation and surface pressure ratio is similar to the chaos response of $\lambda = 400$, although the panel amplitudes have been slightly diminished with a noticeable increase in modal activity. For these four sample locations, the panel is again chaotically fluttering about some mean deflection. This time, the mean deflection and surface pressure has been superimposed with samples of the chaos time history. The low amplitude deflections of $\lambda = 800$ give a clearer representation of this behavior compared with the chaotic response of $\lambda = 400$, although the trends are very much the same. Comparing this mean deflection and mean surface pressures with that of the steady-state equilibrium response of Section 5.4.1, reveals an interesting connection between the mode of the mean deflection and the dominant mode of the steady-state solution. Interestingly, these deflections appear to be nearly identical. This is likely due to surface pressures approaching a linear distribution (when averaged), along with the initialization of the cavity pressure to a zonal average of zones 1 and 3. These two factors would
Figure 5.39

Time history response and frequency spectra of (a-b) $\lambda = 500$, (c-d) $\lambda = 600$, (e-f) $\lambda = 700$, and (g-h) $\lambda = 800$. 
naturally result in a symmetric deflection about the panel mid-chord with a convex leading edge and concave trailing edge, so it is expected that as long as the chaotic deflections are low amplitude the trend will hold true. A sample solution to visualize the associated pressure contour with the oscillatory surface pressure can be found in Figure 5.42, where the resulting pressure fluctuations are quickly cast from the panel surface. Figure 5.43 gives more context to the development of these chaotic flow characteristics. The friction coefficient shows evidence of multiple flow separation locations along the panel length, which is further evidenced by the $u/u_1$ contours (Figure 5.44) and corresponding streamlines.

5.4.4 Feature 4: Time Periodic Solutions

This particular solution ($\lambda = 875$) has been detailed in more depth in Section 4.2.4, however certain aspects of the solution have been shown again for a clear comparison. In
Figure 5.40

(a) Time history slice of chaotic response with arbitrary sample locations for instantaneous (b) panel deformation and (c) surface pressure ratios for $\lambda = 800$ for a laminar boundary layer.
Figure 5.41
(a) 2D and (b) 3D phase plot for $\lambda = 800$ resulting from chaos for a laminar boundary layer.

Figure 5.42
Nondimensional pressure distribution sample (S1) for $\lambda = 800$ resulting from chaos for a laminar boundary layer.

Figure 5.43
Friction coefficient sample (S1) for $\lambda = 800$ resulting from chaos for a laminar boundary layer.
terms of the time history, a limit-cycle instability forms for $\lambda = 875$ after 2 seconds. As seen in Figure 5.45, the time history deflection of 3/4-chord is somewhat similar to that of the inviscid case, with the exception that this time history shows near perfect periodicity, and does not indicate there are influences from other modes and frequencies. Figure 5.46(b) shows the instantaneous deformation over four phases. The phase plots are shown in Figure 5.47.

The pressure ratio (Figure 5.46(a)) shows that along the panel surface the shock has also encountered some diffusion. Through four phases of the oscillation, the pressure does not exhibit a sharp jump or decrease at the leading and trailing edge, nor does the mid-chord reflect a discontinuity as in other inviscid or turbulent predictions of the same $\lambda$. Instead, along with corresponding snapshots of the volume solution in Figure 5.48, the pressure along the panel does not change much as the oscillation continues. Figure 5.49 shows the flow separation and the extent to which the separation has become elongated at several stages of the oscillation. For the phase $\Phi = 180^\circ$, the separation nearly breaks apart.
Figure 5.45

Time history of 3/4-chord displacement for $\lambda = 875$ for a laminar boundary layer.

Figure 5.46

(a) Instantaneous panel deformation and (b) surface pressure ratio for $\lambda = 875$ resulting from a dynamic instability for a laminar boundary layer.
into two separation bubbles due to the influence of the impinging shock. This observation might offer one explanation for the ultimate transition to chaos for $\lambda < 875$. In comparing Figure 5.49 and Figure 5.44, if the separation for $\Phi = 180^\circ$ were to split into two separation bubbles, the impinging shock would prevent a reattachment of the large separation bubble and the number of smaller separation could increase due to resulting irregular fluctuations of the deforming panel. Ultimately, it remains unclear whether chaos is achieved by the separation bubble breaking apart or whether there is a competing effect of structural and fluid instabilities, although the separation bubble is certainly the largest and most obvious differentiating factor between laminar predictions and that of inviscid and turbulent.

### 5.5 Influence of Laminar and Turbulent Boundary Layer on Flutter Predictions
Figure 5.48

Nondimensional pressure distribution for $\lambda = 875$ resulting from a dynamic instability for a laminar boundary layer where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.

Figure 5.49

Friction coefficient for several phase angles resulting from a dynamic instability for a laminar boundary layer.
Figure 5.50

Nondimensional velocity contour for $\lambda = 875$ resulting from a dynamic instability for a laminar boundary layer where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.
Previous sections detailed the similarities and differences in the flow characteristics for inviscid, laminar, and turbulent predictions, but the effect on the bifurcation point and flutter frequency has not been thoroughly discussed to this point. Figure 5.51 contains the bifurcation branches for $\lambda = 100 - 875$. The strong similarities in inviscid and turbulent boundary layer solutions has been previously discussed, but the trend continues for the bifurcation point as well as both sets of solutions share an approximate bifurcation point of $\lambda = 512.5$. As noted in Section 5.4, $\lambda = 100$ and $\lambda = 875$ are the only solutions within the range $\lambda = 100 - 875$ that converge to a steady-state equilibrium and limit-cycle oscillation, respectively. Therefore, these data points have been denoted by dots in Figure 5.51. For intermediate cases that transition to chaos, an average deflection of the chaos is also shown as a reference and is denoted by triangles for each $\lambda$. Figure 5.52 compares the amplitude of the fluttering cases, where there is a 14% increase in amplitude on average for the turbulent boundary layer solutions, and a significant decrease (35%) in the flutter amplitude of the single laminar case in comparison with the inviscid. The Strouhal number, as seen in Figure 5.53, reveals an even closer matching of inviscid and turbulent solutions with an average decrease in $St$ of 1.2%. Conversely, the laminar flutter response exhibits a 7.6% increase in $St$ compared with inviscid. Clearly, a strong correlation exists between inviscid and turbulent solutions in terms of bifurcation point, flutter amplitude, and frequency, among other flow characteristics such as panel deformation and surface pressure capturing in both steady-state equilibrium and time periodic responses. For laminar boundary layers, shock-induced flow separation has translated to significant changes in panel positioning.
and increased diffusion of near wall pressures and surface pressures such that traditional bifurcation behaviors cease to exist, and predictability becomes more difficult to achieve.

![Bifurcation point comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.4$. Triangles indicate the average deflection of the chaos.](image)

**Figure 5.51**

Bifurcation point comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.4$. Triangles indicate the average deflection of the chaos.

### 5.6 $M = 2.0$, Low $Re$ flow with an Oblique Shock Strength of $P_3/P_1 = 1.8$

For low $Re$ flows, the turbulent boundary layer does not capture significant flow alterations that differentiate it from that of the inviscid. If we consider the closeness in the solutions to be solely caused by the lack of flow separation, this may also be the case for high $Re$ flows, where the boundary layer thickness will decrease, and flow separation becomes harder to achieve. It is therefore important to question whether this correlation will hold true for different set of conditions where the boundary layer encounters a more adverse pressure gradient. Looking back to Figure 5.23, although no flow separation exits, a stronger impinging shockwave might be strong enough to induce a flow separation of the turbulent boundary layer. The following solutions presents a preliminary investigation of
Figure 5.52
Flutter amplitude comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.4$.

Figure 5.53
Flutter Strouhal number comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.4$. 
$P_3/P_1 = 1.8$ for $\lambda = 875$ for laminar and turbulent boundary layer solutions. The inviscid solution for this $P_3/P_1$ and $\lambda$ has already been presented in the validation of Section 4.2.3. Figure 5.54 shows the time history displacement of the 3/4-chord for inviscid, laminar, and turbulent boundary layer solutions with this set of conditions. Again, there is a strong similarity in inviscid and turbulent solutions and a transition to chaos for the laminar boundary layer. This chaos behavior is somewhat consistent with feature 2 outlined in Section 5.4.2, although there does not appear to be an increase in the 3/4-chord to some LCO before the transition to chaos. The chaos is also triggered much earlier in the computation compared with that of $P_3/P_1 = 1.4$ predictions.

Figure 5.55 contains the maximum and minimum deflections, Figure 5.56 the flutter amplitudes, and Figure 5.57 the associated flutter frequencies of the 3/4-chord from the previous shock strength compared with the preliminary test case of $P_3/P_1 = 1.8$. For inviscid solutions, the maximum deflection of the 3/4-chord remains steady with a significant increase in the minimum deflection. Similarly, turbulent deflections have shifted down and the amplitude has also become larger. A stronger shock has induced an increase in amplitude of $\approx 37\%$ and $\approx 24\%$ for inviscid and turbulent, respectively, compared with solutions of the same $\lambda$ and $P_3/P_1 = 1.4$. Flutter frequency has also increased by $\approx 29\%$ for both inviscid and turbulent predictions for the stronger shock.

To answer the question of whether a sufficiently strong shock can induce flow separation, Figure 5.58 contains the $u/u_1$ contours for which the streamlines indicate a small flow separation which has indeed been induced by the stronger impinging shockwave. Similarly, the extent of the separation bubble can also be viewed in Figure 5.59, where near the
Figure 5.54

Time history of 3/4-chord displacement for $\lambda = 875$ for (a) inviscid, (b) turbulent, and (c) laminar boundary layer solutions for $P_3/P_1 = 1.8$.

Figure 5.55

Maximum and minimum deflection comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.8$. Data points denote $P_3/P_1 = 1.8$. 

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Figure 5.56
Flutter amplitude comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.8$. Data points denote $P_3/P_1 = 1.8$.

Figure 5.57
Flutter Strouhal number comparison for inviscid, laminar, and turbulent boundary layer predictions for $P_3/P_1 = 1.8$. Data points denote $P_3/P_1 = 1.8$. 

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impingement location, the separation has formed from approximately $0.4 < X/L < 0.5$ the panel length, or which spans $\approx 10\%$ of the panel’s length. For reference, the separation length of $P_3/P_1 = 1.4$ and $\lambda = 875$ for a laminar boundary layer produced a separation that spanned approximately $0.3 < X/L < 0.8$, which is an increase in separation length of nearly $5\times$. Initial observations raised questions to whether the formation of a separation bubble results in a transition to chaotic flow, yet this does not appear to hold true for turbulence modeling where a sufficiently strong shock can induce separation. What now appears to trigger the chaos lies in the extent of the flow separation; the larger the flow separation, the more influence the flow separation will impose on the deformation of the panel. If the flow separation is large enough such that the panel is forced to deform irregularly (away from its primary mode) the flow will surely result in chaos.
Figure 5.58

Nondimensional velocity contour for $\lambda = 875$ and $P_3/P_1 = 1.8$ resulting from a dynamic instability for a turbulent boundary layer where (a) $\Phi = 0^\circ$, (b) $90^\circ$, (c) $180^\circ$, and (d) $270^\circ$ for the oscillation.

Figure 5.59

Friction coefficient for several phase angles resulting from a dynamic instability for a turbulent boundary layer with $P_3/P_1 = 1.8$. 

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CHAPTER 6

CONCLUSIONS

6.1 Conclusions

A coupled FSI solver has been developed and validated by coupling Loci-Chem, a modular, multi-species, chemically reacting, compressible flow solver with the finite-element based structural solver toolkit, MAST. The coupled solver implementation is validated for the prediction of 2D panel instability induced by an initial panel perturbation or shock impingement for a range of subsonic and supersonic Mach numbers, dynamic pressures, shock pressure ratios, and the effect of boundary layer is discussed.

The 2D inviscid flow shows a bifurcation point before which the panel converges to a neutral equilibrium. Subsonic flows show divergence instability (panel reaches steady-state deformation away from neutral position) with either concave or convex panel deflection. Concave deflections are symmetric about the panel midpoint, whereas convex asymmetry can be largely attributed to formation of a normal shock near the location of maximum displacement resulting in a significant drop in surface pressure towards the trailing edge of the panel. Supersonic flows show LCO instability, where the modal deflection activity increases with increasing Mach number. Increasing dynamic pressure results in an increase in equilibrium/LCO amplitude. The 2D panel deflection/LCO amplitude, frequency and bifurcation point predictions compare very well with benchmark results within 7%.
In contrast with uniform flows, panel instabilities induced by the impingement of an oblique shock wave in an inviscid flow show a transition from a steady-state equilibrium to an LCO instability. Shock induced static deformations for a weak shock reveal largely asymmetric modes, with significantly higher deflections at the leading edge, due to higher upstream pressure. For medium and strong shocks, time periodic responses force the trailing edge to alternate from slightly convex to concave, whereas the leading edge simply increases in magnitude of its convex deflection. For weak shocks ($P_3/P_1 = 1.2$), static panel deformations and surface pressure ratios for various dynamic pressure compare very closely with benchmark solutions. There were some differences in panel deformation shape which are likely due to differences in surface pressure due to a sharper capture of the shock discontinuity near the panel mid-chord location. Medium strength shocks ($P_3/P_1 = 1.4$) show large discrepancy in terms of bifurcation location, although differences can likely be attributed to the shock strength’s sensitivity to initial conditions. Far from bifurcation, however, panel deformations, pressure contours, and frequencies are largely consistent. Strong shock strengths ($P_3/P_1 = 1.8$) show characteristically similar deformation and pressure for the fluttering cases to that of the medium shock strength, and the bifurcation prediction is consistent with benchmarks, albeit with a slightly higher flutter amplitude. In general, stronger shocks induce flutter at a lower dynamic pressure and shows higher LCO amplitude than that of weak shocks.

For laminar viscous flows with oblique shock impingement, the characteristics of the flutter shows drastic changes in the overall panel position and pressure distribution compared with that of the inviscid consistent with available benchmark results. A shock-
induced flow separation develops, which significantly increases the panel surface pressure prior to the shock impingement location, forcing the panel position to a lower state. Increase in boundary layer thickness result in a larger separation bubble resulting in a marked difference in panel deformation pattern, increase in LCO amplitude and lowering of LCO frequencies (due to slugging separation bubble shedding) for the same Mach number and dynamic pressure.

Comparison of the inviscid, laminar, and turbulent solutions for $\lambda = 100$–$875$ and $P_3/P_1 = 1.4$ show similarities in behavior for inviscid and turbulent solutions, and unexpected results for the laminar boundary layer. Bifurcation location, equilibrium/LCO amplitudes, and frequencies of fluttering cases reveal strong correlations between inviscid and turbulent (approximately 14% and 1.2% difference, respectively). The instabilities also transition from steady-state equilibrium to LCO near $\lambda = 512$ in both inviscid and turbulent. In addition, there is strong similarity in panel deformations, surface pressure ratios, and pressure contours in the flow field, where the discontinuity-like pressure jump has been preserved. For laminar solutions over the same range, $\lambda = 100$ goes to a steady-state and $\lambda = 875$ is time periodic, but no clear transition is apparent. The static solutions are similar to that of inviscid and turbulent, yet time periodic solutions show much lower LCO amplitudes (35% lower than that of inviscid). Comparison of the panel surface pressures yields an important observation; while inviscid and turbulent simulations maintain discontinuity-like pressure jump along the panel surface, the laminar boundary layers do not. The surface pressure is affected by flow separation (which exists in the entire $\lambda$ range), where pressure ratios encroaching on the impingement location have significantly increased, and the shock
wave is diffused. For intermediate $\lambda$ values with a laminar boundary layer, there is a gradual transition to chaotic flow. Flow separation causes significant alterations in the pressure distribution, panel deformation, and friction coefficient of intermediate cases. Ultimately, the extreme changes in the flow result in chaotic flow patterns caused by the separation breaking apart into two or more separation bubbles. One significant feature of the chaotic flow is the small amplitude, high-frequency oscillations in which the panel oscillates about its mean configuration. This phenomenon has been observed in literature for subsonic Mach numbers with laminar boundary layers, although with some minor differences.

For stronger shocks ($P_3/P_1 = 1.8$) and a fixed $\lambda = 875$, flutter amplitudes increase significantly for both inviscid and turbulent predictions, where the laminar boundary layer again transitions to a chaotic flow whose transient features and development of chaos are not categorically similar to those observed for $P_3/P_1 = 1.4$. Stronger shocks also increase the Strouhal number of the flutter by 29% for non-laminar flows. While a stronger shock induces flow separation for a developing turbulent boundary layer, the bubble occupies approximately 10% of the panel surface compared to 50% coverage of that of the laminar for the same $\lambda$, and the resulting time history response does not trigger chaos. This observation indicates that the flow separation alone does not induce chaos, but rather the extent of the separation bubble.
REFERENCES


