DESIGN, CONSTRUCTION, AND TESTING OF A HIGH ALTITUDE RESEARCH GLIDER

By

Trevor Llewellyn Parker

A Thesis
Submitted to the Faculty of
Mississippi State University
in Partial Fulfillment of the Requirements
for the Degree of Master of Science
in Aerospace Engineering
in the Department of Aerospace Engineering

Mississippi State, Mississippi
December 2010
DESIGN, CONSTRUCTION, AND TESTING OF A HIGH ALTITUDE RESEARCH GLIDER

By

Trevor Llewellyn Parker

Approved:

Keith Koenig
Professor of Aerospace Engineering
(Major Professor)

Calvin R. Walker
Instructor of Aerospace Engineering
(Committee Member)

J. Mark Janus
Associate Professor and Graduate Coordinator for Aerospace Engineering
(Committee Member)

Sarah A. Rajala
Dean of the Bagley College of Engineering
Micro aerial vehicle development and atmospheric flight on Mars are areas that require research in very low Reynolds number flight. Facilities for studying these problems are not widely available. The upper atmosphere of the Earth, approximately 100,000 feet AGL, is readily available and closely resembles the atmosphere on Mars, in both temperature and density. This low density also allows normal size test geometry with a very low Reynolds number. This solves a problem in micro aerial vehicle development; it can be very difficult to manufacture instrumented test apparatus in the small sizes required for conventional testing. This thesis documents the design, construction, and testing of a glider designed to be released from a weather balloon at 100,000 feet AGL and operate in this environment, collecting airfoil and aircraft performance data. The challenges of designing a vehicle to operate in a low Reynolds number, low temperature environment are addressed.
ACKNOWLEDGEMENTS

I would like to thank Dr. Keith Koenig for all the guidance and encouragement he provided. I would also like to thank the NASA Space Grant for funding this project. I would also like to thank Brett Ziegler, Casey Shackelford, and Lorenzo Coley for their assistance with flight testing.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGEMENTS</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>vii</td>
</tr>
</tbody>
</table>

## CHAPTER

<table>
<thead>
<tr>
<th>I. INTRODUCTION</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Previous Work</td>
<td>1</td>
</tr>
<tr>
<td>1.2 FAA Regulations</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II. INITIAL DESIGN</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Airfoil Analysis</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Planform Analysis</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Material Selection</td>
<td>8</td>
</tr>
<tr>
<td>2.4 Surface Actuation</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III. DETAIL DESIGN</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Horizontal Tail Placement</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Static Stability</td>
<td>12</td>
</tr>
<tr>
<td>3.3 Dynamic Stability</td>
<td>14</td>
</tr>
<tr>
<td>3.4 Performance</td>
<td>15</td>
</tr>
<tr>
<td>3.5 CAD Model</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IV. FABRICATION</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>V. FLIGHT TESTING</td>
<td>33</td>
</tr>
<tr>
<td>VI. CONCLUSIONS AND AREAS FOR IMPROVEMENT</td>
<td>42</td>
</tr>
<tr>
<td>6.1 Areas for Improvement</td>
<td>42</td>
</tr>
</tbody>
</table>

iii
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Wortmann FX 63-137 Section Properties, RE=200,000</td>
</tr>
<tr>
<td>2.2</td>
<td>V0 Sizing</td>
</tr>
<tr>
<td>3.1</td>
<td>Dynamic Stability Parameters</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Wortmann FX 63-137 Airfoil Section</td>
<td>4</td>
</tr>
<tr>
<td>2.2</td>
<td>Planform Concepts</td>
<td>6</td>
</tr>
<tr>
<td>2.3</td>
<td>Preliminary CAD Model : V0</td>
<td>7</td>
</tr>
<tr>
<td>2.4</td>
<td>Preliminary CAD Model : V0.1</td>
<td>10</td>
</tr>
<tr>
<td>3.1</td>
<td>Glider Fall and Pull Up</td>
<td>16</td>
</tr>
<tr>
<td>3.2</td>
<td>Initial Wing Design</td>
<td>18</td>
</tr>
<tr>
<td>3.3</td>
<td>Final Wing Design CAD Model</td>
<td>20</td>
</tr>
<tr>
<td>3.4</td>
<td>Wing Root Rib CAD Model</td>
<td>20</td>
</tr>
<tr>
<td>3.5</td>
<td>Vertical Tail CAD Model</td>
<td>22</td>
</tr>
<tr>
<td>3.6</td>
<td>Horizontal Tail Half CAD Model</td>
<td>22</td>
</tr>
<tr>
<td>3.7</td>
<td>Initial Fuselage And Boom CAD Model</td>
<td>23</td>
</tr>
<tr>
<td>3.8</td>
<td>Tail Boom CAD Model</td>
<td>24</td>
</tr>
<tr>
<td>3.9</td>
<td>Fuselage CAD Model</td>
<td>25</td>
</tr>
<tr>
<td>3.10</td>
<td>Parachute Deployment Mechanisms</td>
<td>27</td>
</tr>
<tr>
<td>3.11</td>
<td>Bell Crank System</td>
<td>28</td>
</tr>
<tr>
<td>3.12</td>
<td>Three View plus Isometric View Drawing</td>
<td>29</td>
</tr>
<tr>
<td>4.1</td>
<td>Horizontal Tail Cut Drawing</td>
<td>30</td>
</tr>
<tr>
<td>4.2</td>
<td>Horizontal Tail Cut by Laser Cutter</td>
<td>31</td>
</tr>
<tr>
<td>4.3</td>
<td>Finished Uncovered Airframe</td>
<td>32</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.1 Electric Motor Box and Nose Cone</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>5.2 Glider Prepped For Flight Test</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>5.3 Landing Gear Dolly</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>5.4 High Start Launch Sequence</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>5.5 Hook Installation and Reinforcement</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>5.6 Hook And CG Geometry</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>6.1 Revised Control Arm Model</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>6.2 Effective Jigging Example</td>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>
## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGL</td>
<td>Above Ground Level</td>
</tr>
<tr>
<td>α</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>α₀</td>
<td>Angle of attack at $C_l=0$</td>
</tr>
<tr>
<td>α&lt;sub&gt;stall&lt;/sub&gt;</td>
<td>Stall angle of attack</td>
</tr>
<tr>
<td>CA</td>
<td>Cyano-Acrylate</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Section drag coefficient</td>
</tr>
<tr>
<td>$C_{d\min}$</td>
<td>Minimum drag coefficient</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CG</td>
<td>Center of Gravity</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Section lift coefficient</td>
</tr>
<tr>
<td>$C_{l0}$</td>
<td>Lift coefficient at $\alpha = 0$ degrees</td>
</tr>
<tr>
<td>$C_{la}$</td>
<td>Lift curve slope</td>
</tr>
<tr>
<td>$C_{l\beta}$</td>
<td>Rolling moment due to sideslip</td>
</tr>
<tr>
<td>$C_{l\beta}$</td>
<td>Rolling moment due to sideslip</td>
</tr>
<tr>
<td>$C_{l\max}$</td>
<td>Maximum lift coefficient</td>
</tr>
<tr>
<td>$C_{m\alpha}$</td>
<td>Pitching moment coefficient due to $\alpha$</td>
</tr>
<tr>
<td>$C_{m\delta\epsilon}$</td>
<td>Pitching moment coefficient due to elevator deflection</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>$C_{n\beta}$</td>
<td>Yawing moment due to sideslip</td>
</tr>
<tr>
<td>COTS</td>
<td>Commercial Off The Shelf</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aeronautics Administration</td>
</tr>
<tr>
<td>FAR</td>
<td>Federal Aviation Regulation</td>
</tr>
<tr>
<td>Kerf</td>
<td>The width of material removed by a cutting implement</td>
</tr>
<tr>
<td>Lbf</td>
<td>Pounds Force</td>
</tr>
<tr>
<td>NACA</td>
<td>National Advisory Committee for Aeronautics</td>
</tr>
<tr>
<td>RC</td>
<td>Remote Control</td>
</tr>
<tr>
<td>$s_{sprl}$</td>
<td>Spiral mode root</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>$\omega_{DR}$</td>
<td>Dutch roll natural frequency</td>
</tr>
<tr>
<td>$\omega_{sp}$</td>
<td>Short period natural frequency</td>
</tr>
<tr>
<td>$\zeta_{DR}$</td>
<td>Dutch roll damping ratio</td>
</tr>
<tr>
<td>$\zeta_{ph}$</td>
<td>Phugiod damping ratio</td>
</tr>
<tr>
<td>$\zeta_{sp}$</td>
<td>Short period damping ratio</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

Micro aerial vehicle development and atmospheric flight on Mars are areas that require research in very low Reynolds number flight. Facilities for studying these problems are not widely available. The upper atmosphere of the Earth, approximately 100,000 feet AGL, is readily available and closely resembles the atmosphere on Mars, in both temperature and density. This low density also allows normal size test geometry with a very low Reynolds number. This solves a problem in micro aerial vehicle development; it can be very difficult to manufacture instrumented test apparatus in the small sizes required for conventional testing. To this end, design requirements were formulated for a glider to fly in the upper atmosphere and collect atmospheric data as well as vehicle performance data.

1.1 Previous Work

Similar projects have been conducted. NASA Dryden conducted testing to aid airfoil development in Reynolds Numbers ranging from 200,000 to 700,000. The ARES Mars program is currently in development and a half scale technology demonstrator was tested in 2002. This aircraft had a much larger wing chord and therefore a much larger Reynolds number than the proposed project. There is a group in Canada conducting a very similar mission. However, their glider is much smaller than the one this project proposes and therefore its possible payloads are severely limited [1].
1.2 FAA Regulations

This project is an extension of the balloon satellite program at Mississippi State University. The glider is preferable to the balloon satellite as the satellite is at the mercy of the winds while the glider can control its destination. The glider will be lifted to approximately 100,000 feet AGL by a helium weather balloon and then released and controlled via autopilot. This is well within FAA high altitude balloon regulations, specified in FAR Part 101 [2]. However, current FAA regulations prohibit UAV flight within FAA airspace unless a stringent set of requirements are met. Those requirements specify that the UAV have a certified pilot in the loop, be observed by a chase aircraft that is no further than 3,000 feet vertically and one mile laterally away, and when flown about 18,000 feet the UAV must carry a transponder [3]. This set of requirements is impossible to meet for this mission. In order to perform the mission and satisfy the legal requirements the glider will deploy a parachute at 60,000 feet, the beginning of FAA airspace, and descend as a normal balloon payload.
The project initially intended to utilize a commercial off the shelf remote control glider kit and to modify it to carry the necessary equipment. This was deemed infeasible due to the significant increase in wing loading that this would cause, along with an extremely limited payload capacity. Thus, the need for a purpose designed glider was established. The design should have an empty weight less than ten pounds, conform to all applicable regulations, and be able to carry a variety of payloads.

2.1 Airfoil Analysis

Extensive analysis of airfoil sections was performed using the viscous flow analysis within the program XFLR5 [4]. Over 30 airfoil sections were examined at Reynolds numbers of 20,000 and 200,000 with angle of attack varying between -30 and +30 degrees in 0.1 degree steps. The Reynolds numbers were selected as expected main wing numbers at 100,000 feet and zero feet, respectively. The maximum lift coefficient, drag coefficient at maximum lift, minimum drag coefficient, lift coefficient at minimum drag, minimum pitching moment coefficient, maximum lift to drag ratio, lift and drag coefficients at the maximum lift to drag ratio, zero degree lift coefficient, and the angle of attack for a zero lift coefficient were recorded for each airfoil in an Excel spreadsheet. The airfoil parameter with the best value for a given parameter, e.g. the highest maximum lift coefficient, was highlighted in green while the worst was highlighted in red. It was found that there was very little variation of results between different airfoils for the
Reynolds number of 20,000 and what variation there was may be on the order of the error in the predictions. Using the 200,000 case the Wortmann FX 63-137 was chosen, the parameters for which are found in Table 2.1. This agreed with the choice of the preliminary study conducted by Wahlers [1] and can be viewed in Figure 2.1 [5].

Figure 2.1  Wortmann FX 63-137 Airfoil Section
For the tail surfaces, a NACA 0012 airfoil was chosen. Although it displays some irregularities upon XFLR5 analysis, this airfoil has been used extensively on RC aircraft without ill effect.

### 2.2 Planform Analysis

Several planform types were considered. In order to facilitate airfoil testing at altitude, each concept included a removable test section rather than re-winging the aircraft for each test. Some concepts allowed test section articulation while others would require aircraft to pitch in order to test various test section angles of attack. Pitching the entire aircraft is undesirable due to the additional complexity it would add to the autopilot design as well as negative effects on aircraft performance. Therefore, concepts were deemed viable only if the test section could be articulated independently of the aircraft. Concepts C1, C2, TB1, TB2, and TB3 as found in Figure 2.2 were discarded because it would be prohibitively difficult to articulate only the test section.
Figure 2.2 Planform Concepts

Concept C3 was discarded due to concerns that span-wise flow and tip effects would unduly influence test results. Concepts UC1 and UC2 were eliminated due to concerns of the main wings ability to maintain control as the test section was pitched. Concept CC was chosen over concept TBC because of the decreased lateral stiffness and the increased difficulties of tail surface actuation and attachment to the balloon for nose down release. Concept CC was modified such that the test wing was supported by a boom to distance it from the rest of the aircraft to eliminate blockage effects caused by the fuselage in sideslip. A preliminary CAD model was created to gauge sizing and component locations with an intended construction method of molded composite skins.
with wooden internal bracing. This design was superseded prior to modeling of the test section. It is shown in Figure 2.3 with its sizing listed in Table 2.2.
In order to examine the planform shape of the wing, an Excel worksheet was programmed to optimize the wing shape for minimum Reynolds number at the mean aerodynamic chord, minimum wing weight, and minimum flight velocity by changing the root chord, tip chord, mid chord and span of the wing while satisfying a minimum lift requirement. It was found that the aspect ratio had a negligible effect on wing performance at altitude. Furthermore, given the applied constraints of maintaining a specified amount of lift at the flight conditions, the wing did not become tapered until the span was prohibitively long. Therefore, with the additional advantage of increased construction simplicity, a constant chord wing was chosen. The 14 foot wingspan of the V0 model was considered too long as it would create difficulty in construction, transport, and launch. A maximum span of eight feet was chosen as a reasonable wing size to construct and transport. A minimum aspect ratio of six was chosen, which specified the chord at 16 inches. The incidence of the wing was set to six degrees, the angle of attack for maximum $C_l/C_d$ for the Wortmann FX 63-137, with the intent of having the aircraft trimmed to fly level.

2.3 Material Selection

Remote control models typically use a plastic polymer covering material called MonoKote to form skin over the inner structure of the aircraft. This covering attaches via a heat activated adhesive, and shrinks to a taut surface with further applied heat. Concern that the extreme temperatures found at altitude would cause the material to detach from
the structure or change surface tension initially pushed the design to using molded composite skins. However, molded composites represent a significant expense in time and materials. Fortunately, access was permitted to Gulfstream’s high altitude test chamber. It was found that the cold had very little effect on the material. It does not affect surface tension or adhesion but does make the covering more brittle. However, this was deemed to be acceptable. Therefore, a conventional wooden structure covered with MonoKote was chosen as the construction method. The preliminary model of this design is found in Figure 2.4. The curved shape of this model was deemed too difficult to construct.

2.4 Surface Actuation

Remote control models typically use small electric servo motors to actuate surfaces. These devices are small, require relatively little power, and are fairly inexpensive. In most models the servos are mounted very close to the surface that is actuated in order to have a direct linkage. However, the extreme cold at altitude may hinder their operation. A servo was tested in Gulfstream’s chamber, however due to chamber design the servo was not observed in real time. The servo moved, but it is unknown to what precision it moved or at what speed. Concerned that the servo’s accuracy, precision, and speed would be impacted by the cold, and thereby have a negative affect on aircraft handling, it was determined that the servos should be centrally located in the fuselage, near the other electronics, so that they may be heated.
In consideration of the other systems that would need development, such as the autopilot and pressure measurement system, it was decided that the first glider would be a proof of concept and therefore would omit the test wing and supporting structure.
3.1 Horizontal Tail Placement

Of critical importance to aircraft performance is the size and placement of the horizontal tail. Due to the interactions the tail has with the rest of the aircraft, the placement is invariably a compromise. The longitudinal static stability of the aircraft increases as the surface area of the horizontal tail increases. The longitudinal static stability also increases as the distance between the wing and the horizontal tail increases. These changes will also affect the trim angle required for flight, and the pitch authority of the elevator. However, as the surface area of the tail increases so does the weight. With the increasing weight of the horizontal tail, the weight of the structure that supports it will also increase in order to support the increased load. Similarly, as the tail is moved further aft the length and weight of the structure supporting it also increases [6]. The placement of the tail is an excellent area to apply engineering design optimization, and an optimization code to do precisely that was developed, written in FORTRAN and utilizing Design Optimization Tools (DOT) software. However, this proved unnecessary because of the unique operating conditions of the aircraft. Due to the location of the servos, a linkage system had to be devised to actuate the tail. This linkage system would require a solid push pull type link to span the distance between the servos and the tail. Carbon tubes are commonly used for these applications because of their high degree of stiffness and low weight. Producing these tubes to spec would waste both time and money.
Therefore, commercial off the shelf (COTS) hardware was selected. These products come in limited sizes, the longest being 38 inches. Therefore the distance between the wing and the tail was dictated by the length of the available hardware. The vertical location of the horizontal tail was determined by logic. Mounting on the fuselage or the base of the vertical tail was undesirable due to downwash from the wing and interference from the fuselage. Mounting the tail at some location along the span of the vertical tail complicates the structure of the vertical tail as well as the control of the rudder. Mounting the tail on the top of the vertical tail is structurally simpler to integrate, decreases the influence of the downwash from the main wing and should additionally allow some buffer distance to decrease the influence of the downwash of the test wing that will be added in the future. The drawbacks to this decision are that this increases the length of the elevator control linkage and the vertical tail structure and the attachment structure connecting the vertical tail to the fuselage must handle the additional loads created by the increased moment arm of the horizontal tail. Therefore, with the location of the tail fixed, the size could be determined based on stability requirements.

3.2 Static Stability

The requirement for longitudinal static stability is that $C_{ma}$ must be negative. The static longitudinal stability is calculated by calculating the pitching moment contribution of each aircraft component, in this case the wings, fuselage and tail, and combining them. For this analysis the fuselage was assumed to have not contributed to the pitching moment. In order to calculate the pitching moment contribution of the horizontal tail the downwash from the main wing must be accounted for. This is very difficult to achieve without intensive CFD analysis. Nelson, which was used for the stability analysis,
recommends an approximation [7]. However, DATCOM states that this approximation is valid only when the tail is infinitely far downstream of the wing, a condition that is certainly not met in this case as the wing and tail are a few feet from one another. As an alternative, DATCOM recommends an empirical approximation [8]. This approximation was used; however, due to its empirical nature it describes aircraft with much higher Reynolds numbers than those of the glider in this study. It is unknown how well this will describe the downwash for this case; however, there are no alternatives.

The lateral direction stability was calculated in conjunction with the longitudinal stability. This is due to the horizontal tail being placed on the top of the vertical tail, therefore its position changes based on the design of the vertical tail and this influences the longitudinal stability. The requirements for static lateral and directional stability are that $C_{l\beta}$ is negative and that $C_{n\beta}$ is positive, respectively. These constraints were met with a vertical tail chord of 12 inches and a span of 20 inches, resulting in a $C_{l\beta}$ of -0.029 and a $C_{n\beta}$ of 0.109.

As both tail surfaces would be using the same airfoil section, it makes sense for the chords to be the same in order to simplify part creation. This way, a base NACA 0012 part with a chord of 12 inches can be designed and shared between the tails with slight modifications to tailor them to the specific use. Therefore the only parameter left to specify on the horizontal tail design was the span. A span of 30 inches satisfies the stability requirement, resulting in a $C_{ma}$ of -0.012 per degree. The static margin is used as an indicator of how stable an aircraft is, and is calculated by subtracting the cg location from the neutral point location after both have been normalized by the wing chord. With the non-dimensional neutral point calculated to be 0.47 and the non-dimensional cg location at 0.3 the static margin is 16.9%. Due to the variation in airfoil performance at
lower Reynolds numbers and therefore at higher altitude the static margin increases along with altitude. At 100,000 feet the static margin is 21.7%.

In order for the aircraft to fly trimmed, according to the analysis, a horizontal tail incidence of six degrees was required. However, prior experience in Design Build Fly using the suggested incidence to trim the aircraft resulted in an aircraft that was badly out of trim. Therefore during construction the incidence of the tail was set at zero degrees. However, the elevator is fully flying meaning that the entire horizontal tail will deflect as a unit to effect elevator inputs. This will provide a large amount of elevator control as indicated by a $C_{m\delta e}$ of -1.3. Additionally this allows the incidence of the tail to be adjusted if necessary. It may be also possible to build such adjustment into the autopilot so that the optimal incidence can be used for a given descent condition.

### 3.3 Dynamic Stability

The dynamic stability of the aircraft was determined using analyses from Yechout [9], Raymer [10], and Roskam [11]. The results of the analysis and the associated handling quality levels, based on MIL-F-8785C standards [9], are shown in Table 3.1.
Table 3.1  Dynamic Stability Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sea Level Value</th>
<th>100,000 ft value</th>
<th>Requirement</th>
<th>Handling Quality Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Period Damping Ratio, $\zeta_{sp}$</td>
<td>0.455</td>
<td>0.395</td>
<td>$0.35 \leq \zeta_{sp} \leq 1.3$</td>
<td>1</td>
</tr>
<tr>
<td>Short Period Natural Frequency, $\omega_{sp}$</td>
<td>67.3 rad/s</td>
<td>61.5 rad/s</td>
<td>$\omega_{sp} \geq 1$ rad/s</td>
<td>1</td>
</tr>
<tr>
<td>Phugiod Damping Ratio, $\zeta_{ph}$</td>
<td>0.096</td>
<td>0.064</td>
<td>$\zeta_{ph} &gt; 0.04$</td>
<td>1</td>
</tr>
<tr>
<td>Spiral Mode Root, $s_{sprot}$</td>
<td>-14.0 rad/s</td>
<td>-4.2 rad/s</td>
<td>Design is stable, requirements are based on unstable</td>
<td>1</td>
</tr>
<tr>
<td>Dutch Roll Damping Ratio, $\zeta_{DR}$</td>
<td>0.175</td>
<td>0.054</td>
<td>$\zeta_{DR} \geq 0.02$</td>
<td>2</td>
</tr>
<tr>
<td>Dutch Roll Natural Frequency, $\omega_{DR}$</td>
<td>14.0 rad/s</td>
<td>12.7 rad/s</td>
<td>$\omega_{DR} \geq 0.15$</td>
<td>1</td>
</tr>
<tr>
<td>Dutch Roll, $\omega_{DR} \cdot \zeta_{DR}^*$</td>
<td>2.451 rad/s</td>
<td>0.681 rad/s</td>
<td>$\omega_{DR} \cdot \zeta_{DR}^* \geq 0.4$</td>
<td>1</td>
</tr>
</tbody>
</table>

3.4 Performance

Several performance parameters were calculated for the glider. Of particular interest is the dive and pull up that will be associated with the release from the balloon. After release, the glider must fall straight down until it acquires enough speed to begin flying before pulling up to horizontal flight. Assuming a drop altitude of 100,000 feet, the glider will have to fall for approximately 5,000 feet which will take approximately 18 seconds before beginning the pull up maneuver. If performed at 1.5G the pull up maneuver will require another 5,500 feet and another 14 seconds. If the maneuver is performed at 2G, the entire maneuver will take 8,000 feet rather than the 10,500 feet required for a 1.5G maneuver. In the fall, the glider will reach a maximum speed of
approximately 415 mph or Mach 0.6 which will decrease to approximately 395 mph after pull out. After pull out, the main wing of the glider will have a Reynolds number of approximately 130,000. A 12 inch test wing will have a Reynolds number of approximately 93,000 and a six inch test wing will have a Reynolds number of approximately 47,000. A representation of the fall and 1.5G pull up maneuver is shown in Figure 3.1.

![Figure 3.1 Glider Fall and Pull Up](image)

Previous balloon satellite experiments have released over Birmingham, Alabama and then had to be chased and retrieved. The glider will return to Starkville,
approximately 100 miles away in about 18 minutes and, assuming that it started at 100,000 feet, the glider will arrive in Starkville at 70,000 feet. The lift to drag ratio of the glider is approximately 8.4. The best glide angle for minimum sink varies between -5.7 degrees at 100,000 feet and -4.6 degrees at sea level. The sink rate varies between 1400 feet per minute at 100,000 feet at 170 feet per minute at 1,000 feet. The wing loading of the glider is 1.2 pounds per square foot which is approximately one sixth to one fourth that of full scale gliders. The sea level stall speed is approximately 20 mph.

3.5 CAD Model

With the basic dimensions of the plane decided, the individual components needed to be designed. The plane is to be fabricated out of wood and the design will attempt to leverage that advantageously. Wood is non-homogenous within each piece and varies greatly from piece to piece along with being non-isotropic. Performing structural analysis on assemblies that are made of multiple parts, all made out of wood, would be inordinately difficult and time consuming. Due to the variation in the wood, it may also be worthless. Therefore, careful attention must be paid during the design in order to ensure sufficient structural rigidity. Even so, it may result in some components that are overly rigid and are therefore heavier than necessary and some that may be too weak. It is therefore likely that it will be possible to improve the design after fabrication of the first example is completed. The CAD data was created using Dassault Systemes CATIA V5. Care was taken to ensure that parts were built using the standard aircraft coordinate system which simplifies assembly within the program. Each part was also assigned a material property corresponding to its actual material. This material property changes the visual display of the part and changes the density of the part. Therefore it is possible to
use the CAD model to calculate weights for components and assemblies. Whenever possible, the parts were designed to fit into one another with tabs and slots, thereby using the parts themselves as the assembly jigs.

As the wing is the most critical and the most complex, it was the first component to be designed. In order to make it easier to transport, the wings will be removable. This increases complexity and decreases structural integrity. The first iteration of the wing design used a plywood web in conjunction with balsa flanges in order to create an I-beam structure. The web and a set of secondary flanges would extend into the fuselage in order to be fastened to a common support. This design is illustrated in Figure 3.2

![Initial Wing Design](image)

**Figure 3.2** Initial Wing Design

This design had some faults. It has very little torsional resistance, requires a complex complementary structure in the fuselage and requires care to assemble properly. The next iteration of the wing used a COTS carbon fiber tube as the primary spar. These
tubes are commonly available and are sold with matching fiber reinforced tubes that act as carriers. A 1.5 inch diameter, 48 inch long tube was chosen for this purpose. This results in a simpler installation process as well as a simpler fuselage interface. The wing also utilizes two sets of 1/16 in shear webbing and adds four balsa spars to take over when the carbon spar ends. The shear webs and spars form two opposite facing C-channels. This overlaps the carbon spar in order to ensure rigidity. A leading edge spar, made of balsa, is used to align the ribs at the leading edge during assembly and provide impact resistance. To provide torsional resistance, the forward 30% of both the upper and lower surfaces were sheeted with 1/16 in thick balsa. The sheeting stops at a leading edge cap that aligns with protrusions of the leading edge spar. It is impossible to wrap the sheeting around the leading edge of the wing. This design provides excess material at the leading edge that is sanded to shape the leading edge during assembly. This design used a secondary spar, also a carbon tube, to provide bending resistance to the aft portion of the wing, support the aileron hinges, and act as incidence alignment with the fuselage. A similar carbon rod is used as a torque rod in order to actuate the aileron. The torque rod is carried in plywood bushings keyed into the balsa ribs. The plywood is used because it is much harder and, therefore, will resist wear due to friction. The aileron hinges on a fixed three mm carbon rod. A three mm carbon rod is also used as the trailing edge in order to stiffen it and provide a smooth transition from top surface to bottom surface which will aid in the covering of the wing. This design is shown in Figure 3.3 with the top skin removed for clarity. Incorporating all of these features into the parts is time consuming and results in complicated parts. However the time saved in manufacturing offsets the time spent designing the parts. The root rib of the wing is shown in Figure 3.4, demonstrating the amount of detail in a part.
The vertical and horizontal tails were designed in a similar manner and are shown in Figures 3.5 and 3.6, respectively. The horizontal tail uses four balsa spars, two at the quarter chord and two at the rear. It rotates on a one half inch carbon tube spar that fits into the vertical tail. It uses a six mm carbon rod in parallel to the carbon tube to actuate the rotation. The vertical tail uses six balsa spars, four around the quarter chord and two at the rear. The spars for the vertical tail extend out the bottom and are glued into corresponding slots in the tail boom to affix the tail to the boom. Similar to the wing, it
uses a three mm rod to hinge the rudder. The rudder has a 12 mm carbon rod which is used for actuation.
The initial design for the boom and fuselage used a rectangular prism fuselage section and a six inch diameter circular boom as found in Figure 3.7. The boom used ¼”
by ¼” balsa stringers to carry bending loads and x-braces between each bulkhead at every 90 degrees to carry torsion. Connecting the boom to the fuselage, given the difference in the shapes, was problematic. It was difficult to devise a method to connect the two while effectively transferring the load and not removing usable volume of the fuselage. Therefore the fuselage was made circular to match the boom and the diameter of the boom was increased to eight inches in diameter in order to allow a five inch square cross section for payload in the fuselage. The final boom design can be seen in Figure 3.8.

Figure 3.7 Initial Fuselage And Boom CAD Model
This does present a new problem in the form of the junction between the wing and the fuselage. This requires either the fuselage to be flattened at the junction, the wing root curved, or the wing inset into the fuselage for it to butt against the flat payload wall. Flattening the fuselage and curving the wing moves the attachment point away from the center of the aircraft and will make assembly more difficult. Curving the wing root is structurally complicated. Therefore, insetting the wing is the best solution although it will require some hand fabrication to finish the wing pockets in the fuselage. The plates that butt the wing spar carrier tube have notches in them to allow parachute rigging to be attached to the spar carrier tube and, therefore, the spar, allowing the plane to hang from its CG under parachute. The fuselage has five compartments: two for payload which may
include autopilot functions and batteries, one for servos and the autopilot, one for the parachute, and one for the parachute ejection mechanism. This arrangement can be seen in Figure 3.9.

The first idea for deploying the parachute was to puncture a COTS C02 canister, like those found in BB and paint ball guns. The canister would be propelled by the expanding gas and be ejected from a PVC pipe open to the atmosphere. The canister would drag the deployment bay hatch and the parachute bay hatch behind it, separating them from the fuselage. To facilitate this, the deployment hatch and the parachute hatch are held on with ¼ inch diameter neodymium N50 magnets. If necessary a drogue parachute would be included; however, the hatches themselves are probably sufficient drogues to deploy the main parachute. In order to facilitate this arrangement, the top of
the payload frame that separates the deployment bay and the parachute bay has a small relieved section to allow a connecting cable to pass between the two bays. Similarly, the fuselage plates that support the spar carrier tube are notched to allow the parachute rigging to fasten around the spar carrier tube. This allows the parachute to connect to a significant structural member and also allows the glider to hang from its CG while descending under the parachute. Two mechanisms were built to attempt to puncture the CO\textsubscript{2} canister. The first uses a spring loaded ram inside an aluminum cylinder. A screw on the bottom is tightened to compress the spring. A rod is inserted through the cylinder and prevents the ram from releasing. The screw is removed and the mechanism actuated by removing the rod. This mechanism is on the right in Figure 3.10. Another mechanism was devised using a mousetrap with an aluminum plate fitted to the arm. The plate struck a ram, forcing it against the diaphragm on the CO\textsubscript{2} canister. This mechanism is shown on the left in Figure 3.10. Neither of these systems worked, failing to provide enough pressure to burst the diaphragm of the CO\textsubscript{2} canister. The manufacturer was unable to provide a spec for the minimum burst pressure required. Other methods that have considered but not yet implemented are using a COTS solid rocket motor in lieu of the CO\textsubscript{2} canister or utilizing an additional set of magnets with reversed polarity and mechanically switching between attracting and repelling pairs to eject the hatch.
In order to actuate the tail surfaces, and as seen in Figure 3.11, a bell crank system was devised. This allows the pushrods from the servos to run along the floor and then the bell cranks connect to a new pushrod to actuate the mechanism. There are three bell cranks at the back of the tail, one for the vertical tail, one for the horizontal tail, and one for the balloon release. The bell cranks pivot on a carbon rod.
The CAD model was used to create production drawings for each assembly. These drawings included several views of the assembly along with labeled parts and the quantity required to create each assembly. The final model contained a total of 713 parts comprised of 183 unique parts. A three view plus isometric view drawing of the complete aircraft is shown in Figure 3.12.
Figure 3.12 Three View plus Isometric View Drawing
CHAPTER IV

FABRICATION

The wooden parts of the glider were cut from blanks using a laser cutter owned by the Architecture department at Mississippi State University. This device operates much like a printer and interfaces with AutoCAD. In order to create the files necessary for part creation, the assembled CAD model was broken down and the parts were placed on blanks the size of the piece of wood that the parts would be cut from. An extra sketch was created on most parts to label the part. These labels would be engraved on the part during production. A drawing was then created of the part layouts and color coded based on whether the machine was to ignore the line, cut the line, or engrave the line as shown in Figure 4.1. The laser produces a very small kerf, so the part sizes were not adjusted to account for it. This example in Figure 4.1 is shown being cut by the laser in Figure 4.2.

![Figure 4.1 Horizontal Tail Cut Drawing](image_url)
Figure 4.2  Horizontal Tail Cut by Laser Cutter

It required approximately seven hours of machine time to produce all the parts necessary to build one glider. This represents a significant time savings; to manufacture the parts by hand to the same precision would require a tremendous amount of time.

Fabrication was completed in Patterson Labs. To produce an assembly, the production drawing is referenced and all the parts listed in it are removed from their respective sheets. The parts are then dry fit together. This allows each part to move a small amount as the rest of the parts are added, facilitating the self jigging action. Once all the appropriate parts are fitted together the alignment is checked to ensure that it is satisfactory. After the fit is deemed satisfactory, the assembly is bonded together using CA adhesive. The most time intensive and difficult process of fabrication is sheeting the wing surfaces. Unfortunately, there is no simple method of improving this process. After an assembly is completed it is then covered with MonoKote, provided it does not have to be attached to any other assemblies. A single horizontal stabilizer half can be assembled in an hour, and covered in less than one more hour. The CAD model predicted that a single horizontal stabilizer would weigh 2.34 oz not including covering, the actual stabilizer halves weighed 2.24 oz and 2.26 oz each; not including covering. This indicates that the predictions are fairly accurate. On larger assemblies it will under predict the weight as the model does not include the adhesive used to join the parts. The weight of
the entire finished, covered glider is 7.3 lbf. The CAD Model predicted a weight of 6.4 lbf without covering. The covering added approximately 0.8 lbf. Due to the design it requires additional weight to be balanced. This weight is added in the form of the payload. The completed, uncovered airframe is shown in Figure 4.3.

Figure 4.3  Finished Uncovered Airframe
CHAPTER V
FLIGHT TESTING

With the glider constructed, the next step was to flight test it. A full scale, drop from 100,000 feet test is not prudent. Furthermore, the project does not yet have an autopilot. Therefore, initial testing will be performed under traditional RC control at low altitudes. This presents a problem in launching. The glider was designed to be dropped, however there is not a suitable structure from which to drop it. Tethering a balloon at an altitude appropriate for dropping would require notifying the FAA and the test site is close to the local airport so the altitude permitted would be limited. Additionally, as this type of testing is likely to happen often as new components are designed and integrated, tethering a balloon would present a hassle as well as the expense of the helium to fill it and possibly the cost of the balloon. Therefore, there is a requirement for the ability to launch it from the ground.

To foster this, a replacement nose cone was designed along with the supporting structure for an electric motor as shown in Figure 5.1. The motor used was a Neu 1506-3y-5s geared in-runner brushless three phase motor, connected to a JETI electronic speed controller and powered by a 20 cell Elite 1500 mAh NiMh battery pack. This system was connected to an Aeronaut CAM 16x13 carbon folding propeller. These components were chosen because they were left over from a DBF plane from several years ago and powered a plane of similar weight.
Flight test equipment included the power plant and propulsion battery pack, a GPS balloon payload module, two SP2600 five cell battery packs to power the servos, a Futaba 2.4GHz RC receiver, and a GoPro HD Hero video camera. The assembled glider is shown in Figure 5.2 in preparation for the test flight.
The first flight test was attempted as a hand launch, which is very common for RC aircraft that do not have landing gear. This method employs an assistant to run and throw the aircraft in lieu of a takeoff run. Unfortunately, this launch did not get the aircraft above the stall speed and with the low power fraction of the motor and the short distance to the ground there was not enough time to accelerate past the stall speed. This resulted in the glider effectively falling to the ground, slowed by the lift that was produced, and very slightly damaging the airframe in the form a cracked section of stringer on the very bottom of the fuselage. It also broke the propeller, which was replaced with an APC 19x8 glass filled nylon propeller. When tested, this new propeller produced an average five
pounds of thrust. Although unsuccessful as a flight test, it demonstrated that the aircraft was balanced appropriately as it remained level in its descent.

The next step was to provide the glider with enough velocity to take off. A dolly was investigated as an idea to allow the glider to get up to flight velocity on the ground in the manner of landing gear and then take off, leaving the dolly behind. The dolly was constructed using landing gear left over from COTS RC aircraft and is shown in Figure 5.3.

Figure 5.3  Landing Gear Dolly

Before this was attempted with the high altitude glider, it was tested with a COTS RC glider. Weight was added to the dolly to simulate the load that would be placed on it by the research glider. A length of elastic tubing was connected to the dolly and to a stake in the ground. This tubing was then stretched in order to provide the propulsive force for the dolly. This proved ineffective as the elastic tubing did not provide enough force to accelerate the dolly to the research glider’s flight speed. As an alternative to the elastic tubing the dolly was connected to a van with rope. The rope was run through two stakes
in the ground with eyelets in order to effect two 90 degree turns of the rope so that the van would be driving in the opposite direction of the glider and not be interfering with it. This allows controlled acceleration to a chosen speed. This method presented a few problems. The dolly had passive steering and due to the rough terrain of the testing field the dolly did not track in a straight line. Also, the glider did not exhibit good control at low speeds under tow, particularly in roll as a crosswind would attempt to flip the glider off the dolly. Any structure added to the dolly to impede this action would also impede takeoff. The takeoff from the dolly once flight speed was achieved was abrupt and somewhat unpredictable. Therefore, the dolly was determined to not be an acceptable method of launching the research glider.

A common method for launching RC gliders is called a high start. This method employs a section of elastic material and rope attached to the glider and to the ground. The attachment to the glider is usually achieved by a loop in the rope slipped over a hook attached to the glider. The elastic is stretched until it achieves a tensile force between three and five times the weight of the glider. The glider is held at an extreme angle, typically around 45 degrees and then released. An example of this can be seen in Figure 5.4 using the COTS glider previously mentioned.
In order to employ this method a hook needed to be added to the glider. Because of the weight of the glider, the hook would need to be strong enough to resist the launch load of 36 to 65 pounds. Therefore, the foremost concern in adding the hook was its attachment to existing structure to ensure it would not be ripped from the aircraft. To achieve this, the hook was located directly below the wing spar attachment point. This point is also the nominal CG location; most RC gliders have the hook placed on or slightly ahead of the CG. The hook was made out of G10 epoxyglas, as were the bulkhead and floor reinforcement pieces. It was initially planned to reinforce both sides of the floor and further brace the floor against the wing spar tube and bulkhead. However, after installing the hook and reinforcing the bulkhead and bottom of the floor it was deemed strong enough and that further reinforcement was unnecessary. This is shown in Figure 5.5.
The high start was constructed using 30 feet of 5/8” OD 1/8” ID elastic spear gun tubing and 200 feet of #36 masons’ line connected with steel rings and attached to a stake in the ground. The line was loaded to 50 pounds of tension and connected to the glider which was held at a 45 degree angle. Upon release the glider maintained this angle and did not climb. Power was applied and the glider released from the hook. The glider was brought under control but by this time it was off course and at low altitude so the test was aborted. A lack of familiarity with the aircraft, coupled with the sensitivity of the fully flying elevator resulted in a less than perfect landing. There was a moderate amount of damage which required a few hours to fix. The most notable of this was that two of the balsa spars that connected the vertical tail to the fuselage cracked at the point of
connection. This was repaired and reinforced. Reasoning that the launch angle was the culprit, the second attempt used a much shallower launch angle. However, upon release the glider pitched up to an extreme angle. There was not enough time or altitude for the aircraft to recover into normal flight and as a result experienced a hard impact which caused the tail structure to break off from the fuselage.

Upon examination, it was determined that in the concern of positioning the hook for structural integrity, the position of the hook in relation to the actual center of gravity was over looked. The attachment point of the hook and the line varies between 0.38 inches and 0.56 inches behind the actual center of gravity depending on the angle of the line. This in itself is not very bad, and is due to the CG being shifted forward by the flight test equipment carried. More importantly, the hook is directly below the nominal design CG point. The problem lies in the fact that the attachment point is between 6.5 and 6.8 inches below the center of gravity. This geometry is shown in Figure 5.6.
This produces a large amount of pitching moment about the CG due to the launch load. A tensile load of 50 pounds applied to the aircraft with a 15 degree line angle will produce a 27 ft*lbf moment. The horizontal tail, after accelerating under a 50 lbf load for one second and at five degrees, will only produce a five ft*lbf moment to counter. Therefore the hook must be relocated. Because of the depth of the fuselage, the normal RC glider method of mounting it at or slightly ahead of the CG will not work. Instead, it must be at the front of the plane, similar to a full scale glider.
CHAPTER VI
CONCLUSIONS AND AREAS FOR IMPROVEMENT

The glider demonstrated controlled flight, if only for a few seconds. It did this while carrying approximately 4.5 pounds of payload. By this measure, the project has been successful. However, in order for the project to continue, it must be possible to test the aircraft for more than a few seconds at a time. This then, will require a revised method of launching the aircraft for testing. This may be accomplished by moving the launch hook and using the high start as planned, by fitting a more powerful motor, or possibly by launching it from a moving vehicle. Given the uncertainty of the aerodynamics at these Reynolds number, controlled testing with a full scale model at the flight conditions is warranted and recommended.

6.1 Areas for Improvement

The control arms used for all surface actuation were cut out of aircraft plywood. Due to variations in the density of the wood, the kerf on some parts was larger than others. As a result, some of the rotating parts had extra clearance and there was play in the movement. Furthermore, due the laminated nature of the wood, side loads such as those produced when attaching clevises can cause the arms to break. These problems can be solved by making them out of a different material, such as aluminum, carbon fiber, or glass filled nylon composite. The area around the main rotation holes in the bell crank should be thickened in order to limit off axis rotation as shown on the right in Figure 6.1.
The attachment point of the vertical tail is sufficient for normal operation. However, hard impacts such as those the glider may experience when descending under the recovery parachute onto less than perfect terrain could damage it as it was damaged during flight testing. This can be prevented by reinforcing the balsa spars that form the attachment mechanism with flat pieces of carbon fiber. There are COTS pieces that are six mm by one mm that would provide adequate reinforcement with minimum increased weight. The corresponding slots in the vertical tail ribs would need to be deepened to accommodate the extra thickness.

It is possible to reduce the weight of the aircraft. The payload area, currently made out of birch aircraft plywood, could be made out of light plywood. Light plywood is roughly half the weight of the birch plywood. This substitution would save roughly eight ounces of weight. The X braces in this area may also be removed, with little ill effect. It may also be possible to reduce the number of bulkheads and to reduce the
number of stringers in the boom. The current boom and fuselage is impressively stiff, and some of that stiffness may be sacrificed for weight loss if it is deemed necessary.

The self-jigging of the parts works very well, but can be improved. Some tabs, such as those on the bulkheads could benefit from reshaping in order to lock the pieces together rather than just aligning them as in the current design. The tabs on the payload sections are good examples to follow, shown in Figure 6.2. Pieces for rib alignment should include break away pieces on either end in order to capture the end ribs rather than require them to be pressed into the jig.

Figure 6.2   Effective Jigging Example
There are a few other minor improvements that could be made. The ¼-20 nylon bolts that hold the first three hatches on are overkill and can be replaced with neodymium magnets. During testing the rear two hatches did not separate, indicating that this is a sufficient attachment method. The ¼-20 nylon nuts fasted in the wing root ribs need to be reinforced to prevent damage from over tightening of the bolts. The ailerons and elevator should have spacers on the hinges to prevent travel along the hinge axis. The fuselage can be slightly flattened at the wing junction to eliminate the need for hand fabrication at the junction. An external location for switches to activate servos, autopilot, payload modules, etc should be added to allow these functions to be activated and deactivated externally after the aircraft is assembled. This location should have a cover to prevent accidental activation or deactivation of the switches. Finally, tabs should be added to the first bulkhead in the fuselage to allow mounting structures to be added to support things mounted in the nose, such as a nose mounted camera.

6.2 Future Work

Given the problems with aircraft trim using conventional analysis in the past, the lack of a good downwash model, and the future addition of a test wing, full scale trim testing should be conducted. The current airframe could be used for this. It is possible to retrofit the existing airframe with a new tail to replace the broken one. The hook in the current airframe could be modified to be a pivot point by drilling a hole in it, and inserting a metal tube and bearings. Additionally, a complementary fixture for the ground test vehicle could be constructed. This fixture should provide the ability to roll and yaw. This would allow testing at speed to determine trim, downwash effects, control surface effectiveness, parachute deployment, autopilot development, etc without undue risk to the
airframe. This would be especially useful when the test wing is added and may become a prerequisite for each new test wing or change in payload.
REFERENCES


Atmospheric Model
by Trevor Parker

This model calculates atmospheric properties, based on altitude ASL. The equations used can be found in many aircraft texts, notably Mechanics of High Performance Aircraft by Vihn. The values calculated are for a STANDARD DAY.

Atmospheric Constants:

\[ R_{\text{air}} := \frac{g}{s^2 R} \]
\[ \gamma := 1.4 \]
\[ \rho_1 := 0.000704030 \frac{\text{slug}}{\text{ft}^3} \]
\[ \rho_2 := 7.656 \times 10^{-5} \frac{\text{slug}}{\text{ft}^3} \]

\[ T_{\text{sl}} := 518.69 \text{K} \]
\[ T_1 := 389.99 \text{K} \]
\[ T_2 := 216.65 \text{K} \]
\[ T_3 := 228.65 \text{K} \]
\[ h_1 := 36089 \text{ft} \]
\[ \beta_1 := -0.0019812 \frac{K}{\text{ft}} \]
\[ \beta_2 := 0.0003048 \frac{K}{\text{ft}} \]
\[ \beta_3 := 0.00085344 \frac{K}{\text{ft}} \]
\[ h_2 := 65617 \text{ft} \]
\[ h_3 := 104987 \text{ft} \]
\[ \rho_{100k} := 3.2 \times 10^{-5} \frac{\text{slug}}{\text{ft}^3} \]
\[ \rho_{80k} := 4.0084 \times 10^{-2} \frac{\text{kg}}{\text{m}^3} \]
\[ \rho_{20k} := 5474.89 \text{Pa} \]
\[ \rho_{216.65} := 216.65 \text{K} \]
\[ \rho_{228.65} := 228.65 \text{K} \]
\[ h_{104987} := 104987 \text{ft} \]

Altitude Dependent Functions:

\[ T(h) := \begin{cases} T_{\text{sl}} - \beta_1 h & \text{if } h < h_1 \\ T_2 & \text{if } h_1 \leq h \leq h_2 \\ T_2 + \beta_2 (h - h_2) & \text{if } h_2 < h < h_3 \\ T_3 + \beta_3 (h - h_3) & \text{if } h_3 < h \end{cases} \]

\[ \rho(h) := \begin{cases} \rho_{100k} & \text{if } h < h_1 \\ \frac{\rho_1}{T_{\text{sl}}} \left( \frac{T(h)}{T_{\text{sl}}} \right) & \text{if } h_1 \leq h \leq h_2 \\ \frac{\rho_2}{T_2} \left( \frac{T(h)}{T_2} \right) & \text{if } h_2 < h < h_3 \\ \frac{\rho_3}{T_3} \left( \frac{T(h)}{T_3} \right) & \text{if } h_3 < h \end{cases} \]

\[ \mu(h) := \begin{cases} 10^{-7} \frac{\text{ft} \cdot \text{s}}{\text{lbf}} & \text{if } h = 0 \\ 3.737 & \text{if } 0 < h < h_1 \\ -5 \frac{\text{h}}{\text{ft}} - 2.10 \frac{\text{h}}{\text{ft}} + 3.7368 & \text{if } 0 < h < h_1 \\ 2.969 & \text{if } 0 < h < h_1 \\ 3.737 & \text{if } h = 0 \\ -11 \frac{\text{h}}{\text{ft}} + 2.10 \frac{\text{h}}{\text{ft}} - 5 \frac{\text{h}}{\text{ft}} \text{ otherwise} \end{cases} \]

Callable Functions:

\[ P(h) := \rho(h) R_{\text{air}} T(h) \]

\[ \alpha(h) := \sqrt{\frac{\gamma P(h)}{\rho(h) R_{\text{air}}}} \]

\[ V(M, h) := M \alpha(h) \]

\[ R_d(h, V, L) := \frac{\rho(h) V L}{\mu(h)} \]

\[ \alpha(h) := \frac{\rho(h)}{\rho_{104987}} \]

\[ M(V, h) := \frac{V}{\alpha(h)} \]

\[ \sigma_{\text{ratio}}(h) := \frac{\rho(h)}{\rho_{104987}} \]

---

49
Airfoil Data

Wing Data for WFX 63-137

tocwing := 0.1367

...Thickness over chord ratio for wing

\( e_O := 0.80 \)

...Oswalds efficiency factor approximation

0 ft:

\( C_{macw} := -0.2 \)

...XFLR5 at RE 200K, M=0

\( C_{L,max} := 1.7 \)

...Maximum lift coefficient, XFLR5 at RE 200K, M=0

\( C_{L,max_flap} := 1.59 \)

...Maximum lift coefficient with flap deployed

\( \alpha_0 := -7 \text{deg} \)

...Angle of attack for zero lift, XFLR5 at RE 200K, M=0

\( \alpha_{max} := 9.8 \text{deg} \)

...Stall AoA, XFLR5 at RE 200K, M=0

\[
C_{1\alpha w} = \frac{C_{L,max}}{\alpha_{max} - \alpha_0} = 0.10119 \text{deg}^{-1}
\]

...Lift curve slope

Tail Data for NACA 0012

0 ft:

\( C_{1\alpha H} = 0.1 \text{deg}^{-1} \)

...Thin Airfoil Theory

\( C_{1\alpha V} = 0.1 \text{deg}^{-1} \)

...Thin Airfoil Theory

\( toc_{tail} := 0.12 \)

...Thickness over chord ratio for horizontal tail

\( toc_{vert} := 0.12 \)

...Thickness over chord ratio for vertical tail

Fuselage Data

\( C_{macf} := 0 \)

....Fuselage Pitching moment coefficient

\( C_{1\alpha f} = 0 \text{deg}^{-1} \)

...Fuselage lift curve slope

\( \alpha_{0f} := 0 \text{deg} \)

...Fuselage zero lift angle of attack
**Flight Conditions**

\[ W_t := 12\text{lb} \]  
... weight of the airplane, use best estimate or weight approximation sheet

\[ h_{cr} := 90000\text{ft} \]  
...Cruise altitude, Starville is at 330ft ASL, test flights usually occur at ~50-200ft AGL.

\[ \rho_{cr} := \rho_{100k} \]  
...Cruise density

\[ V_{cr} := 270\text{knot} \]  
...Cruise velocity

\[ M_{cr} := 0.5 \]  
...Cruise Mach number

[\[ \alpha_{\text{flight}} := 0\text{deg} \]  
...angle of attack

\[ h_{to} := 330\text{ft} \]  
...Altitude for takeoff. NOTE these are for STANDARD DAYS, look up equivalent density altitude for your location

\[ W_{to} := W_t \]  
...Takeoff weight

\[ \mu_r := 0.05 \]  
...rolling friction coefficient

\[ T_{to} := 0\text{ozf} \]  
...Takeoff Thrust

\[ p_a := 0\text{W} \]  
...Power Available

\[ T_W := \frac{T_{to}}{W} \]  
...Takeoff Thrust to weight ratio

\[ \beta_{to} := 1 \]  
...Weight fraction. 1 for takeoff (fully loaded)

\[ n_{to} := 1 \]  
...Load factor
Main Wing Properties

\[ b := 8\text{ft} \quad \ldots \text{Wing span} \]

\[ c_r := 16\text{in} \quad \ldots \text{Wing root chord} \]

\[ c_t := 16\text{in} \quad \ldots \text{Wing tip chord} \]

\[ \lambda := \frac{c_t}{c_r} \quad \lambda = 1 \quad \ldots \text{Taper ratio} \]

\[ c := \frac{2}{3} c_r \left(1 + \lambda - \frac{\lambda}{\lambda + 1}\right) \quad c = 16\text{in} \quad \ldots \text{Wing mean Geo/Aero chord} \]

\[ S := \frac{b}{2} \left(\frac{c_t + c_r}{2}\right)^2 \quad S = 1536\text{in}^2 \quad \ldots \text{Wing area} \]

\[ WS := \frac{W_t}{S} \quad WS = 18\text{ozf}\text{ft}^2 \quad \ldots \text{Wing loading} \]

\[ A_R := \frac{b^2}{S} \quad A_R = 6 \quad \ldots \text{Aspect ratio} \]

\[ \Lambda := 0\text{deg} \quad \ldots \text{Wing leading edge sweep angle} \]

\[ \Gamma := \frac{1}{2} \tan \theta_{\text{wing}} \left(c_t - c_r\right) \quad \Gamma = 0\text{deg} \quad \ldots \text{calculated dihedral for a flat top surface} \]

\[ i_w := 6\text{deg} \quad \ldots \text{Wing incidence angle} \]

\[ Q_{\text{wing}} := 1.4 \quad \ldots \text{Wing Interference Factor: 1.0 for well filleted wings, 1.1-1.2 for most designs, 1.4 for no smoothing} \]

\[ X_{\text{ac - c}} := 0.25 \quad \ldots \text{Wing Aerodynamic center as percent mean chord, Assume 0.25 unless otherwise known!} \]

\[ k := \frac{1}{\pi q_{\text{Y-A}}} \quad k = 0.066 \quad \ldots \text{VIRTUAL distance from wing root c/4 to the fuselage centerline, positive for low wing, negative for high wing} \]
Main Wing Properties Cont

\( h_w := 6 \text{in} \)  

\[ \gamma_{gew} := \frac{b}{6} \left( 1 + 2\lambda \right) = 24 \text{in} \]  

\[ \Lambda_{e, 0.25ct} := \text{atan} \left[ \frac{\tan(A) \left( \frac{b}{2} + \frac{c_t}{4} \right)}{\frac{b}{2}} \right] = 4.764 \text{deg} \]  

\[ x_{le, 0.25ct} := \frac{0.5b}{\tan(90\text{deg} - \Lambda_{e, 0.25ct})} = 4 \text{in} \]  

\[ \Lambda_{0.25} := \text{atan} \left[ \frac{0.5b}{0.25c_t - x_{le, 0.25ct}} \right] + 90\text{deg} = 0 \text{deg} \]  

\[ \Lambda_{e, 0.5ct} := \text{atan} \left[ \frac{\tan(A) \left( \frac{b}{2} + \frac{c_t}{2} \right)}{\frac{b}{2}} \right] = 9.462 \text{deg} \]  

\[ x_{le, 0.5ct} := \frac{0.5b}{\tan(90\text{deg} - \Lambda_{e, 0.5ct})} = 8 \text{in} \]  

\[ \Lambda_{0.5} := \text{atan} \left[ \frac{0.5b}{0.5c_t - x_{le, 0.5ct}} \right] + 90\text{deg} = 0 \text{deg} \]  

\( \Delta cf := 0 \)  

\( \delta_y := 0 \text{deg} \)
Fuselage and Layout Properties

$l_f$ := 72 in  
... length of fuselage

$D_f$ := 8 in  
... Fuse max diameter

$\rho_f := \frac{D_f}{2}$  
... Fuse max radius

$A_x := \pi \rho_f^2$  
... Fuse max cross sectional area, note modeling as circle, replace with more accurate area if/when available

$Q_{fuse} := 1.0$  
... Fuselage Interference Factor: 1.0 for well filleted, smoothed fuselages, 1.1-1.2 for most designs, 1.4 for no smoothing, greater for protrusions

$X_{cg_c} := 0.3$  
... CG location as percent chord of the wing, 0.3 is a good starting point

$X_{acw_ach} := 30.625$ in  
... Distance between AC of wing and AC of horizontal tail (quarter chord to quarter chord)

$X_{acw_acv} := 30$ in  
... Distance between AC of wing and AC of vertical tail (quarter chord to quarter chord)

$X_{acw_acf} := \text{0}$_m  
... Distance between AC of wing and AC of fuselage, positive for fuse ac fore of cg

$X_{cg_ach} := X_{acw_ach} - c \left( X_{cg_c} - X_{ac_c} \right)$  
... Distance between the ac of the horizontal tail and the cg

$X_{cg_ach} = 29.825$ in

$X_{cg_acv} := X_{acw_acv} - c \left( X_{cg_c} - X_{ac_c} \right)$  
... Distance between the ac of the vertical tail and the cg

$X_{cg_acv} = 29.2$ in
Fuselage and Layout Properties Cont

\( s_\text{f} := 130h^2 \)  
...Fuselage profile area

\( l_{\text{cg}} := 33\text{in} \)  
...Distance from nose to cg

\( h_\text{fl} := 8\text{in} \)  
...Height (profile thickness) of the fuselage at the \( l/4 \) point

\( b_\text{fl} := 8\text{in} \)  
...Width (spanwise thickness) of the fuselage at the \( l/4 \) point

\( h_{\text{fmax}} := D_f \)  
...Max height of fuselage

\( b_\Omega := 8\text{in} \)  
...Width (spanwise thickness) of the fuselage at the \( 3l/4 \) point

\( h_\Omega := 8\text{in} \)  
...Height (profile thickness) of the fuselage at the \( 3*l/4 \) point

\( X_{1f} := 38\text{in} \)  
...Location of maximum negative derivative of fuselage profile

\( x_{\text{of}} := \left( 0.378 + 0.527 \frac{X_{1f}}{l_f} \right) l_f \)  
...Location of loss of potential flow along the fuselage profile

\( s_o := \pi (3\text{in})^2 = 28.274\text{in}^2 \)  
...Fuselage cross sectional area at \( x_{\text{of}} \)
Horizontal Tail Properties

\[ b_H := 30\text{in} \quad \text{...Horizontal tail span} \]

\[ c_{th} := 12\text{in} \quad \text{...Horizontal root chord} \]

\[ c_{th} := 12\text{in} \quad \text{...Horizontal tip chord} \]

\[ \lambda_h := \frac{c_{th}}{c_{th}} \quad \lambda_h = 1 \quad \text{...Horizontal taper ratio} \]

\[ c_H := \frac{2}{3}
\left[ 1 + \lambda_h - \frac{\lambda_h}{\lambda_h + 1} \right] \quad \text{...Horizontal mean Geo/Aero chord} \]

\[ S_H := b_H \left( \frac{c_{th} + c_{th}}{2} \right) \quad S_H = 360\text{in}^2 \quad \text{...Horizontal tail area} \]

\[ \frac{S_H}{S} = 23.437\% \quad \text{...Horizontal tail area as a percent of wing area, should be \sim15-25\% for conventional designs} \]

\[ A_{RH} := \frac{b_H^2}{S_H} \quad A_{RH} = 2.5 \quad \text{... Horizontal aspect ratio} \]

\[ k_H := \frac{1}{\pi \sqrt{O^2 A_{RH}}} \quad k_H = 0.159 \]

\[ z_h := 26\text{in} \quad \text{...Vertical distance between the wing mac and h.tail mac, tail above wing is positive} \]

\[ \Lambda := 5\text{deg} \quad \text{...Horizontal leading edge sweep angle} \]

\[ \Lambda_{0.25} := 0 \quad \text{...Horizontal sweep at 0.25 chord line} \]

\[ \Lambda_{0.5} := 0 \quad \text{...Horizontal sweep at 0.5 chord line} \]
Horizontal Tail Properties Cont.

\( i_h := 6 \text{deg} \)  
...Horizontal incidence angle

\( Q_{\text{tail}} := 1.2 \)  
...Tail Interference Factor: 1.0 for well filleted tails, 1.1-1.2 for most designs, 1.4 for no smoothing

\( x_{\text{Hac,c}} := 0.25 \)  
...Horizontal Aerodynamic center as percent mean chord, Assume 0.25 unless otherwise known!

\( \eta_h := 1 \)  
...This is the ratio of dynamic pressure over the horizontal tail to the dynamic pressure over the wing. Assumed to be 1, unless otherwise known.

\[ V_H := \frac{X_{cg,ach} S_H}{S_c} \]  
...Horizontal tail volume ratio

\( c_{re} := 1 \cdot c_{th} \)  
...Elevator root chord

\( c_{te} := 1 \cdot c_{th} \)  
...Elevator tip chord

\( b_e := b_H \)  
...Elevator span

\[ s_E := (b_e - 5\text{in}) \left( \frac{c_{re} + c_{te}}{2} \right) \]  
...Elevator Area

\[ \frac{s_{E}}{s_{H}} = 0.833 \]  
...Elevator area to full stab area ratio

\[ \tau_e = -10.644 S_{E}^{4} + 17.578 S_{E}^{3} - 10.7 S_{E}^{2} + 3.584 S_{E} + 0.001 \]  
...Elevator effectiveness parameter. Set to 1.0 for full flying tail (stabilator). Curve from NACA ARR No. L4116, fit using Excel. Note that SE in this case is the elevator area AFT of the hinge line (does not include counterbalances, counterbalances may need to count as negative effective area).
**Vertical Tail Properties**

\[ b_v := 20\text{in} \]

...Vertical tail span

\[ c_{rv} := 12\text{in} \]

...Vertical root chord

\[ c_{tv} := 12\text{in} \]

...Vertical tip chord

\[ \lambda_v := \frac{c_{tv}}{c_{rv}} = 1 \]

...Vertical taper ratio

\[ c_v := \frac{2}{3} c_{tv} \left( 1 + \lambda_v - \frac{\lambda_v}{\lambda_v + 1} \right) \]

...Vertical mean Geo/Aero chord

\[ S_v := b_v \left( \frac{c_{tv} + c_{rv}}{2} \right) \]

...Vertical tail area

\[ S_v = 240\text{in}^2 \]

\[ \frac{S_v}{S} = 15.625\% \]

...Vertical tail area as a percent of wing area, should be ~15-20% for conventional designs

\[ \Lambda_{Rv} := \frac{b_v^2}{S_v} \]

\[ \Lambda_{Rv} = 1.667 \]

...Vertical aspect ratio

\[ k_v := \frac{1}{\pi \rho_0 A_{Rv}} \]

\[ k_v = 0.239 \]

\[ \Lambda_v := 15\text{deg} \]

...Vertical leading edge sweep angle

\[ \Lambda_{0.5v} := 0 \]

...Vertical sweep at 0.5 chord line

\[ i_v := 0\text{deg} \]

...Vertical incidence angle

\[ Z_{acv} := \frac{b_v \left( 1 + 2 \lambda_v \right)}{3 \left( 1 + \lambda_v \right)} = 10\text{in} \]

...height of vertical tail ac above aircraft centerline

\[ h_v := \frac{Z_{acv} + D_f}{2} \]

...Vertical tail Interference Factor: 1.0 for well filleted tails, 1.1-1.2 for most designs, 1.4 for no smoothing

\[ Q_{vert} := 1.2 \]

...Vertical Aerodynamic center as percent mean chord, Assume 0.25 unless otherwise known!

\[ X_{Vac_c} := 0.25 \]

...This is the ratio of dynamic pressure over the vertical tail to the dynamic pressure over the wing. Assumed to be 1, unless otherwise known.

\[ \eta_v := 1 \]

**Wing**

\[
Re_{wing} := \frac{\rho(330\text{ft}) V_{cr} \cdot \mu}{(330\text{ft})} \quad Re_{wing} = 3.828 \times 10^6 \\
M_{cd0E} := M(V_{cr}, h_{cr}) \quad M_{cd0E} = 0.414
\]

\[
Cf_{wing} := \begin{cases} 
0.455 & \frac{\log(Re_{wing})}{2.58} \left(1 + 0.144 M_{cd0E}\right)^2 \quad \text{if } Re_{wing} > 0.5 \times 10^6 \\
1.328 & \frac{Re_{wing}}{0.5} \quad \text{otherwise}
\end{cases}
\]

\[
F_{starwing} := 1 + 3.3 \cdot toc_{wing} - 0.008 toc_{wing}^2 + 27 toc_{wing}^3 \quad F_{wing} := \left( F_{starwing} - 1 \right) \cos\left(\Lambda_{0.5} \right)^2 + 1
\]

\[
Cd0_{wing} := Cf_{wing} \cdot F_{wing} \cdot Q_{wing} \left( \frac{2S}{S} \right) 
\]

\[
Cd0_{wing} = 0.015
\]

**Fuselage**

\[
\lambda_f := \frac{1}{\left( \frac{4}{\pi} \cdot A_x \right)^{0.5}} \quad Re_{fus} := \frac{\rho(330\text{ft}) V_{cr} \cdot \mu_f}{(330\text{ft})} \quad Re_{fus} = 1.722 \times 10^7 \\
F_{fus} := 1 + \frac{2.2}{\lambda_f^{1.5}} - \frac{0.9}{\lambda_f^{3.0}} \quad F_{fus} = 1.08
\]

\[
S_{wetfus} := \pi \cdot D_f \cdot A_x \\
Cf_{fus} := \begin{cases} 
0.455 & \frac{\log(Re_{fus})}{2.58} \left(1 + 0.144 M_{cd0E}\right)^2 \quad \text{if } Re_{fus} > 0.5 \times 10^6 \\
1.328 & \frac{Re_{fus}}{0.5} \quad \text{otherwise}
\end{cases}
\]

\[
Cd0_{fus} := Cf_{fus} \cdot F_{fus} \cdot Q_{fus} \left( \frac{S_{wetfus}}{S} \right)
\]

\[
Cd0_{fus} = 3.664 \times 10^{-3}
\]

59
CD0 Estimation Continued

**Horizontal Tail**

\[
R_{\text{tail}} := \frac{\rho \sqrt{V_c e H}}{\mu(330 \text{ft})} \quad R_{\text{tail}} = 2.871 \times 10^6 \quad F_{\text{startail}} := 1 + 3.52 \cdot \omega_{\text{tail}} \quad F_{\text{tail}} := (F_{\text{startail}} - 1) \cos^2(L) + 1
\]

\[
C_{f_{\text{tail}}} := \begin{cases} 
\frac{0.455 \log(R_{\text{tail}})}{2.58} \left(1 + 0.144 M_{\text{cd0E}}\right)^2 & \text{if } R_{\text{tail}} > 0.5 \times 10^6 \\
1.328 & \text{otherwise}
\end{cases}
\]

\[
C_{d0_{\text{tail}}} := C_{f_{\text{tail}}} F_{\text{tail}} Q_{\text{tail}} \left(\frac{S_H}{S}\right) \quad C_{d0_{\text{tail}}} = 3.006 \times 10^{-3}
\]

**Vertical Tail**

\[
R_{\text{vert}} := \frac{\rho \sqrt{V_c e v}}{\mu(330 \text{ft})} \quad R_{\text{vert}} = 2.871 \times 10^6 \quad F_{\text{starvert}} := 1 + 3.52 \cdot \omega_{\text{vert}} \quad F_{\text{vert}} := (F_{\text{starvert}} - 1) \cos^2(L) + 1
\]

\[
C_{f_{\text{vert}}} := \begin{cases} 
\frac{0.455 \log(R_{\text{vert}})}{2.58} \left(1 + 0.144 M_{\text{cd0E}}\right)^2 & \text{if } R_{\text{vert}} > 0.5 \times 10^6 \\
1.328 & \text{otherwise}
\end{cases}
\]

\[
C_{d0_{\text{vert}}} := C_{f_{\text{vert}}} F_{\text{vert}} Q_{\text{vert}} \left(\frac{S_V}{S}\right) \quad C_{d0_{\text{vert}}} = 2.004 \times 10^{-3}
\]

**Total Aircraft**

\[
C_{d0} := C_{d0_{\text{fuse}}} + C_{d0_{\text{wing}}} + C_{d0_{\text{tail}}} + C_{d0_{\text{vert}}} \quad C_{d0} = 0.024
\]

\[
C_{D0} := C_{d0}
\]
3D Effects on Lift and Drag

\[ C_L = C_{L\alpha w}(\alpha_{light} + i_w - \alpha_0) \]
\[ C_L = 1.315 \]

\[ C_{L\alpha w3D} = \frac{C_{L\alpha w}}{1 + k \cdot C_{L\alpha w}} \]
\[ C_{L\alpha w3D} = 0.073 \frac{1}{\text{deg}} \]

\[ C_{L3D} = C_{L\alpha w3D}(\alpha_{light} + i_w - \alpha_0) \]
\[ C_{L3D} = 0.95 \]

\[ C_{L\alpha H3D} = \frac{C_{L\alpha H}}{1 + k_H \cdot C_{L\alpha H}} \]
\[ C_{L\alpha H3D} = 0.052 \frac{\text{deg}}{} \]

\[ C_{L\alpha V3D} = \frac{C_{L\alpha V}}{1 + k_V \cdot C_{L\alpha V}} \]
\[ C_{L\alpha V3D} = 0.042 \frac{\text{deg}}{} \]

\[ C_D = C_{D0} + k \cdot C_L^2 \]
\[ C_D = 0.139 \]

\[ C_{D3D} = C_{D0} + k \cdot C_{L3D}^2 \]
\[ C_{D3D} = 0.084 \]

\[ C_{L\text{star}} = \sqrt{\frac{C_{D0}}{k}} \]
\[ C_{L\text{star}} = 0.6 \]

\[ E_{\text{max}} = \frac{1}{2 \sqrt{k \cdot C_{D0}}} \]
\[ E_{\text{max}} = 12.562 \]

\[ \frac{C_L}{C_D} = 9.488 \]

\[ \frac{C_{L3D}}{C_{D3D}} = 11.344 \]
Longitudinal Static Stability

\[ C_{L0w} := C_{L0w3D}(\alpha_w + i_w) \]
\[ C_{L0f} := C_{L0f3D}(\alpha_f) \]
\[ e_0 := 2C_{L0w}k \]

...Reference Lift Coefficient

...Fuselage reference lift coefficient

...Reference Downwash angle

Downwash gradient (DATCOM empirical method):

\[ K_a := \frac{1}{A_R} - \frac{1}{1 + A_R} \]
\[ K_\lambda := \frac{10 - 3\lambda}{7} \]
\[ K_h := \frac{1 - \frac{z_h}{b}}{\frac{1}{3} \left( \frac{2X_{cg,ach}}{b} \right)} \]

\[ K_a = 0.121 \quad K_\lambda = 1 \quad K_h = 0.855 \]

\[ \frac{d\varepsilon}{d\alpha} := 4.44 \left( K_a K_\lambda K_h \sqrt{\cos \left( \frac{\alpha_0,25}{\text{deg}} \right)} \right)^{1.19} \]
\[ \frac{d\varepsilon}{d\alpha} = 0.299 \]

...Downwash derivative wrt to angle of attack

Note, this calculation of the downwash gradient comes from DATCOM. It is an empirical equation, which means that it is based on large aircraft so it may not accurately represent low Reynolds number aircraft.

\[ \frac{d\varepsilon}{d\alpha}_{\text{nelson}} := \frac{2C_{L0w3D}}{\pi A_R e_0} \]
\[ \frac{d\varepsilon}{d\alpha}_{\text{nelson}} = 0.555 \]

...Note: this equation that Nelson gives for the downwash gradient for the tail assumes, according to DATCOM, that the tail is infinitely far downstream from the wing. I disagree. Use of this equation will result in lower static margin (less stable) than the use of the DATCOM empirical method.
Longitudinal Static Stability Cont.

Wing contribution to pitching moment coefficient:

\[ C_{m_{0w}} := C_{m_{acw}} + C_{L_0w} \left( X_{cg_c} - X_{ac_c} \right) \]

\[ C_{m_{0w}} = -0.152 \]

....Contribution of the wing to the level flight pitching moment coefficient

\[ C_{m_{\alpha w}} := C_{L_\alpha w3D} \left( X_{cg_c} - X_{ac_c} \right) \]

\[ C_{m_{\alpha w}} = 0.0037 \frac{1}{\text{deg}} \]

....Contribution of the wing to pitching moment coefficient wrt to changes in angle of attack

Horizontal tail contribution to pitching moment coefficient:

\[ C_{m_{0h}} := \eta_h V_H C_{ldH3D} \left( \epsilon_1 + i_w - i_h \right) \]

\[ C_{m_{0h}} = 0.165 \]

....Contribution of the horizontal tail to the level flight pitching moment coefficient

\[ C_{m_{\alpha h}} := -\eta_h V_H C_{ldH3D} \left( 1 - \frac{c_{d\alpha}}{c} \right) \]

\[ C_{m_{\alpha h}} = -0.016 \frac{1}{\text{deg}} \]

....Contribution of the horizontal tail to pitching moment coefficient wrt to changes in angle of attack

Fuselage contribution to pitching moment coefficient:

\[ C_{m_{0f}} := C_{m_{acf}} + C_{L_0f} \left( X_{cg_c} - X_{ac_c} - \frac{X_{acw_acf}}{c} \right) \]

\[ C_{m_{0f}} = 0 \]

....Contribution of the fuselage to the level flight pitching moment coefficient

\[ C_{m_{\alpha f}} := C_{L_\alpha f} \left( X_{cg_c} - X_{ac_c} - \frac{X_{acw_acf}}{c} \right) \]

\[ C_{m_{\alpha f}} = 0 \frac{1}{\text{deg}} \]

....Contribution of the fuselage to pitching moment coefficient wrt to changes in angle of attack
Longitudinal Static Stability Cont.

Combined Pitching moment coefficients:

\[ C_{m0} := C_{m0w} + C_{m0h} + C_{m0f} \quad C_{m0} = 0.013 \]

\[ C_{m\alpha} := C_{m\alpha w} + C_{m\alpha h} + C_{m\alpha f} \quad C_{m\alpha} = -0.012 \, \text{deg}^{-1} \quad \ldots \text{Cm}\alpha \text{ must be negative for static stability} \]

Total Pitching Moment:

\[ C_m := C_{m0} + C_{m\alpha} \alpha_{\text{light}} \quad C_m = 0.013 \]

Stick Fixed Neutral Point:

\[ X_{np,.c} := X_{ac,.c} - \frac{C_{m\alpha}}{C_{L\alpha w3D}} + \eta_h \, V_{HI} \frac{C_{L\alpha H3D}}{C_{L\alpha w3D}} \left( 1 - \delta_{\alpha f\alpha} \right) \]

\[ X_{np,.c} = 0.469 \]

Static Margin:

\[ SM := X_{np,.c} - X_{cg,.c} \quad SM = 16.914\% \]

Full Body trim angle:

\[ \alpha_t := \frac{-C_{m0}}{C_{m\alpha}} \quad \alpha_t = 1.011 \, \text{deg} \]

Pitching moment due to elevator deflection:

\[ C_{m\alpha\epsilon} := -C_{L\alpha H3D} \eta_h \, V_{HI} \frac{S_H}{S} \left( \frac{X_{cg,ach}}{c} \right) \tau_e \quad C_{m\alpha\epsilon} = -1.309 \]

\[ \ldots \text{Pitching moment due to elevator deflection. Typical value } \sim -1. \text{ More negative indicates more control.} \]
Lateral Directional (Roll and Yaw) Stability

Side Force

\[ d \sigma_\beta := 1 - \eta_v \left( 0.724 + \frac{3.06 S_v}{S} \frac{S_v}{1 + \cos(\Lambda_{0.25})} + 0.4 z_w + 0.009 \Lambda_R \right) \]

\[ d \sigma_\beta = 0.022 \]

...sidewash derivative with respect to sideslip angle, analogous to \( \frac{dc_{\beta \alpha}}{dx} \) usually small

\[ C_{y\beta v} := -c_L \alpha \frac{S_v}{S} \left( 1 - d \sigma_\beta \right) \eta_v \]

\[ C_{y\beta v} = -0.37 \]

\[ \ldots \text{Contribution of the vertical tail to the side force coefficient, negative indicates positive stability because a positive sideslip will produce a force out the left wing, or negative y axis} \]

From Airplane Design part VI by Roskam

\[ l_v := X_{acw, acv} \]

\[ z_v := h_v \]

\[ C_{yr} := -2 c_y \beta_v \left( l_v \cos(\alpha_{f \text{light}}) + z_v \sin(\alpha_{f \text{light}}) \right) / b = 0.231 \]

\[ C_{y\beta w} = -0.00573 \left[ \Gamma \right] \]

\[ \ldots \text{Side force due to yaw rate} \]

\[ \kappa_y := \begin{cases} 
-1.85 \left( \frac{z_w}{D_f 0.5} \right) & \text{if } \frac{z_w}{D_f 0.5} < 0 \\
1.5 \left( \frac{z_w}{D_f 0.5} \right) & \text{otherwise}
\end{cases} \]

\[ \frac{z_w}{D_f 0.5} = 0 \quad \kappa_y = 0 \]

\[ C_{y\beta f} = -2 \kappa_y \frac{S_v}{S} = 0 \]

\[ C_{y\beta} = C_{y\beta w} + C_{y\beta v} + C_{y\beta f} = -0.37 \]

\[ \ldots \text{Fuselage contribution to side force due to sideslip} \]

\[ \ldots \text{Side force due to sideslip} \]
Lateral Directional (Roll and Yaw) Stability Cont.

Rolling Moment

Requirement for lateral stability: \( C_{l\beta} < 0 \)

From Airplane Design part VI by Roskam

\[ C_{Lw} = C_{L3D} \]

\[ C_{l\beta}_{CL - A0.5} = 0 \text{ deg}^{-1} \]

\[ \frac{A_R}{\cos(A_{0.5})} - 6 \]

\( K_{MA} = 1.1 \)

...Wing sweep contribution to the rolling moment due to sideslip. Figure 10.20 in Roskam VI. Dependent on taper ratio, 1/2 chord sweep angle, and aspect ratio.

\( K_f = 1.0 \)

...Fuselage correction factor. Figure 10.22 in Roskam VI.

\[ C_{l\beta}_{AR} = -0.00175 \]

...Wing aspect ratio contribution. Figure 10.23 in Roskam VI.

\[ C_{l\beta}_{\Gamma} = -0.00022 \]

...Wing geometric dihedral contribution. Figure 10.24 in Roskam VI.

\( K_{M\Gamma} = 1.1 \)

...Compressibility correction to dihedral contribution. Figure 10.25 in Roskam VI. Will be 1 for low speed a/c.

\[ D_{avg} = D_f = 0.203 \text{ m} \]

\[ \Delta C_{l\beta}\Gamma = -0.0005 A_R \left( \frac{D_{avg}}{b} \right)^2 = -2.083 \times 10^{-5} \]

...Fuselage induced effect on wing height.

\[ \Delta C_{l\beta,zw} = \sqrt{0.042 A_R \frac{z_w}{D_{avg} b}} \]

...Contribution of wing height
Lateral Directional (Roll and Yaw) Stability Cont.

Rolling Moment Cont.

\( \varepsilon_y := 0 \text{deg} \)  
...Wing twist, measured as the angle between the root zero lift line and the tip zero lift line.

\[ \Delta C_{I\beta \text{t}, \tan \Lambda, 0.25} = -0.000035 \]  
...Wing twist contribution factor. Figure 10.26 in Roskam VI.

\[ C_{I\beta_{wf}} = C_{Lwfr}(C_{I\beta_{CL, 0.5}, K_M, K_T} + C_{I\beta_{CL, AR}}) + \Gamma \left( C_{I\beta_{FL}} + \Delta C_{I\beta_{\Gamma}} + \Delta C_{I\beta_{ZW}} + \varepsilon_{t, \tan \Lambda, 0.25} \right) \]  
...Wing fuselage contribution to the rolling moment due to side slip

\[ C_{I\beta_{wf}} = -1.663 \times 10^{-3} \]  
...Wing fuselage contribution to the rolling moment due to side slip

\[ C_{I\beta_{\Gamma}} = \frac{-1}{6} C_{Lw3D \Gamma} \left( \frac{1 + 2 \lambda}{1 + \lambda} \right) \]  
...Contribution of dihedral to the rolling moment due to side slip coefficient

\[ C_{I\beta_{\lambda}} = -2 C_{L3D} \left( \frac{x_{aw}}{b} \right) \sin(2 \lambda) \]  
...Contribution of wing sweep to the rolling moment due to side slip coefficient

\[ \chi_v := \frac{1}{3} b \left( \frac{1 + 2 \lambda}{1 + \lambda} \right) \]  
\( \chi_v = 0.178 \text{m} \)  
...Estimation for the height of the vertical tail ac

\[ C_{I\beta_v} = \frac{\chi_v}{b} C_{y\beta} \]  
\( C_{I\beta_v} = -0.027 \)  
...Contribution of the vertical tail to the rolling moment due to side slip coefficient

\[ C_{I\beta} = C_{I\beta_{wf}} + C_{I\beta_v} + C_{I\beta_{\lambda}} + C_{I\beta_{\Gamma}} \]  
...Rolling moment due to side slip coefficient

Requirement for lateral stability : \( C_{I\beta} < 0 \)
Lateral Directional (Roll and Yaw) Stability Cont.

Rolling Moment Cont.

\[ B = \left( 1 - M_r^2 \cos^2 \left( \alpha_{0,25} \right) \right)^{1/2} \]

\[ \text{Cl}_{\text{rCL-CL0M}} = 0.275 \quad \text{...Roskam VI, figure 10.41} \]

\[ \frac{\text{Cl}_{\text{rCL-CLM}}}{\text{Cl}_{\text{rCL0M0}}} = \frac{1 + A_R \left( 1 - B^2 \right)}{2 B \left( A_R B + 2 \cos \left( \alpha_{0,25} \right) \right)} + \frac{A_R B + 2 \cos \left( \alpha_{0,25} \right) \tan \left( \alpha_{0,25} \right)^2}{A_R B + 4 \cos \left( \alpha_{0,25} \right)} \quad \text{...Roskam VI} \]

\[ \Delta \text{Cl}_{\Gamma} = 0.083 \left( \frac{\pi A_R \sin \left( \alpha_{0,25} \right)}{A_R + 4 \cos \left( \alpha_{0,25} \right)} \right) \quad \text{...Change in rolling moment moment due to yaw rate due to dihedral} \]

\[ \Delta \text{Cl}_{\epsilon_2} = 0.0035 \quad \text{...effect of wing twist on Clr, Roskam VI, figure 10.42} \]

\[ \Delta \text{Cl}_{\alpha_{\delta f}} = 0 \quad \text{...effect of flaps on Clr, Roskam VI, figure 10.43} \]

\[ \text{C}_{\text{rw}} = C_{L3D} \text{Cl}_{rCL-CL0M} + \Delta \text{Cl}_{\Gamma} \Gamma + \Delta \text{Cl}_{\epsilon_2} \epsilon_2 + \Delta \text{Cl}_{\alpha_{\delta f}} \alpha_{\delta f} \quad \text{...Wing contribution to rolling moment due to yaw rate} \]

\[ \text{C}_{\text{rv}} = -\frac{2}{b^2} \left( l_y \cos(\alpha_{\text{fligth}}) + z_y \sin(\alpha_{\text{fligth}}) \right) \left( z_y \cos(\alpha_{\text{fligth}}) - l_y \sin(\alpha_{\text{fligth}}) \right) C_y \beta_v \quad \text{...Vertical tail contribution to rolling moment due to yaw rate} \]

\[ \text{C}_{\text{r}} = \text{C}_{\text{rw}} + \text{C}_{\text{rv}} = 0.31 \quad \text{...rolling moment due to yaw rate} \]
Lateral Directional (Roll and Yaw) Stability Cont.

**Yawing Moment**

Requirement for directional stability $c_{n\beta} > 0$

\[
c_{n\beta v} := -\frac{X_{\text{cg, acv}}}{b} \cdot c_{y v}
\]

$|c_{n\beta v}| = 0.112$

...Contribution of the vertical tail to the yawing moment coefficient. Note that this is the strongest contributor to $c_{n\beta}$ and must be positive.

\[
k_{\beta} := 0.3 \frac{l_{f}}{l_{f}} + 0.75 \frac{h_{\text{max}}}{l_{f}} - 0.105 \quad \Rightarrow \quad k_{\beta} = 0.116
\]

...empirical factor, valid for $\frac{l_{f}}{h_{\text{max}}} \geq 3.5$

\[
c_{n\beta f} := -k_{\beta} \frac{S_{b} l_{f}}{S_{b}} \frac{h_{\eta}}{h_{\eta}} \left( \frac{b_{\theta}}{b_{\eta}} \right)^{3} \quad \Rightarrow \quad c_{n\beta f} = -7.353 \times 10^{-3}
\]

...Contribution of the fuselage to the yawing moment coefficient. Note its dependency on $k_{\beta}$ and the validity of the calculation thereof.

$c_{n\beta w} := 0$

$c_{n\beta} := c_{n\beta w} + c_{n\beta v} + c_{n\beta f}$

$|c_{n\beta}| = 0.105$

...Wing contribution to the yawing moment due to sideslip. Zero except for wings at high angles of attack.

$c_{n w_{12}} := -0.25$

$c_{n w_{0}} := -0.35$

$c_{n w} := C_{n w_{12}} C_{L3D}^{2} + C_{n w_{0}} C_{D0}$

$c_{n w} = -0.234$

$c_{n w} := -\frac{2}{b_{2}} \left( l_{v} \cos(\alpha_{\text{light}}) + z_{v} \sin(\alpha_{\text{light}}) \right)^{2} c_{y v}$

$c_{n w} = -0.072$

$c_{n w} := c_{n w_{12}} + c_{n w_{0}} = -0.306$

...Wing contribution to yawing moment due to yaw rate, Roskam VI

...Vertical tail contribution to yawing moment due to yaw rate Roskam VI

...Yawing moment due to yaw rate

---

69
Dynamic Stability

From Yechout

\[ \text{mass} := \frac{W_{to}}{g} \]

\[ \sigma_{\text{bar}} := c \]

\[ U_{1} := V_{cr} = 310.71 \text{ mph} \]

\[ \rho_{1} := \rho_{cr} \]

\[ M_{1} := M_{cr} \]

\[ q_{\text{bar}1} := \frac{1}{2} \rho_{1} V_{1}^{2} \]

\[ C_{L\alpha\text{hat}} = C_{L\alpha w3D} = 0.073 \text{ deg}^{-1} \]

\[ C_{D1} := C_{D3D} = 0.084 \]

\[ C_{m\alpha\text{hat}} = C_{m\alpha} = -0.708 \]

\[ C_{mq} := -2C_{L_{H3D}}n_{h}V_{H}\frac{X_{acw,ach}}{q_{\text{bar}}} = -5.012 \]

\[ C_{madot} := -2C_{L_{H3D}}d_{\alpha\text{ch}}n_{h}V_{H}\frac{X_{acw,ach}}{q_{\text{bar}}} = -1.499 \]

\[ Z_{\alpha} := \frac{-q_{\text{bar}1}S}{\text{mass}} \left( C_{L\alpha\text{hat}} + 2C_{D1} \right) \]

\[ R_{y\text{bar}} := 0.38 \quad \ldots \text{Pitch Radius of gyration. Raymer} \]

\[ R_{x\text{bar}} := 0.25 \quad \ldots \text{Roll Radius of gyration. Raymer} \]

\[ R_{z\text{bar}} := 0.39 \quad \ldots \text{Yaw Radius of gyration. Raymer} \]
Dynamic Stability Cont.

- Pitching moment of inertia. Use eq from Raymer or calculate with CAD.
  \[ I_{yy} := \frac{l_{\text{mass}}^2 r_{\text{ybar}}^2}{4} = 0.657 \text{m}^2 \text{kg} \]

- CAD

- Rolling moment of inertia. Use eq from Raymer or calculate with CAD.
  \[ I_{yx} := 0.0101 \text{kg m}^2 = 0.01 \text{m}^2 \text{kg} \]

- CAD

- Yawing moment of inertia. Use eq from Raymer or calculate with CAD.
  \[ I_{xz} := \frac{b + l_f}{2} \frac{l_{\text{mass}}^2 r_{\text{xbar}}^2}{4} = 0.942 \text{m}^2 \text{kg} \]

- CAD

- CAD

\[ M_{\alpha} := q_{\text{bar1}} S c_{\text{bar}} \frac{2}{l_{yy}} C_{m\theta} \]

\[ M_{\dot{\alpha}} := q_{\text{bar1}} S c_{\text{bar}} \frac{2}{2 l_{yy} U_1} C_{m\dot{\theta}} \]

\[ C_{L1} := C_{L3D} \]

\[ C_{Lu} := \frac{M_1^2 \cos(\Lambda_{0.25})^2 C_{L1}}{1 - M_1^2 \cos(\Lambda_{0.25})^2} = 0.317 \]

- Change in CL wrt perturbed velocity. Assume to be zero for subsonic flight.

- Thrust due to speed at steady state

\[ C_{Tx1} := 0 \]

- Change in thrust due to speed when perturbed

\[ C_{Txu} := 0 \]

- Change in CD wrt perturbed velocity. Assume to be zero for subsonic flight.

\[ C_{Du} := 0 \]
Dynamic Stability Cont.

**Short Period**

\[ \omega_{hsp} := \sqrt{\frac{Z\alpha M_q}{U_1}} - M_{\alpha} = 67.345 \frac{1}{s} \]

\[ \zeta_{sp} := \frac{M_q + Z\alpha M_{\alpha} + M_{\alpha \dot{\theta}}}{2 \omega_{hsp}} = 0.455 \]

\[ T_{po_{sp}} := \frac{2\pi}{\omega_{hsp} \sqrt{1 - \zeta_{sp}^2}} = 0.105s \]

\[ T_{sp} := \frac{4}{\zeta_{sp} \omega_{hsp}} = 0.13s \]

**Phugoid**

\[ \omega_{ph} := \sqrt{\frac{g q_{bar1} S (C_{L1} + 2 C_{L1})}{U_1^2 \text{mass}}} = 0.181 \frac{1}{s} \]

\[ \zeta_{ph} := \frac{(C_{Du} + 2 C_{D1} - C_{Txu} - 2 C_{Tx1}) q_{bar1} S}{2 \text{mass} U_1 \omega_{ph}} = 0.097 \]

\[ T_{po_{ph}} := \frac{2\pi}{\omega_{ph} \sqrt{1 - \zeta_{ph}^2}} = 34.942s \]

\[ T_{sp} := \frac{4}{\zeta_{ph} \omega_{ph}} = 229.014s \]

\[ L_{\beta} := \frac{C_{1\beta} q_{bar1} S \cdot b}{l_{xx}} = \frac{-53.318}{2} \frac{1}{s} \]

\[ N_{\gamma} := \frac{C_{n\gamma} q_{bar1} S \cdot b^2}{2 l_{zz} U_1} = \frac{-4.83}{1} \frac{1}{s} \]

\[ Y_{\beta} := \frac{C_{y\beta} q_{bar1} S}{\text{mass}} = 10.711 \frac{m}{s} \]

\[ L_{r} := \frac{C_{1r} q_{bar1} S \cdot b^2}{2 l_{xx} U_1} = \frac{5.061}{1} \frac{1}{s} \]

\[ N_{\beta} := \frac{C_{n\beta} q_{bar1} S \cdot b}{l_{zz}} = 188.847 \frac{1}{s} \]

\[ Y_{r} := \frac{C_{y\gamma} q_{bar1} S \cdot b}{2 \text{mass} U_1} = 0.059 \frac{m}{s} \]
Dynamic Stability Cont.

Spiral Mode

\[ s_{sp} = \frac{N\beta L - \beta N r}{L \beta + N\beta \left( \frac{I_z}{I_x} \right)} = -13.303 \frac{1}{s} \]

...Spiral root. Negative indicates a stable spiral mode. It is "ok" to have an unstable spiral mode, check the time to double amplitude and the time constant to ensure acceptability.

\[ \tau_s := \frac{L \beta + N\beta \left( \frac{I_z}{I_x} \right)}{N\beta L - \beta N r} = -0.075s \]

...Spiral mode time constant

\[ T_{2s} := \frac{L \beta \ln(2)}{N\beta L - \beta N r} = -0.053s \]

...Spiral mode time to double amplitude

Dutch Roll

\[ \omega_{hD} = \left[ \left( \frac{Y \beta N r + N\beta (U_1 - Y_1)}{U_1} \right) \right]^\frac{1}{2} = 13.753 \frac{1}{s} \]

...Dutch roll natural frequency

\[ \zeta_D := \frac{N_r + \frac{\sqrt{\beta}}{U_1}}{2 \omega_{hD}} = 0.178 \]

...Dutch roll damping ratio

\[ \zeta_D \omega_{hD} = 2.453 \frac{1}{s} \]

\[ T_{poD} := \frac{2\pi}{\omega_{hD} \sqrt{1 - \zeta_D^2}} = 0.464s \]

...Dutch roll period of oscillation

\[ T_{SD} := \frac{4}{\zeta_D \omega_{hD}} = 1.63s \]

...Dutch Roll settling time
Glider Fall and Pull Up

\[ L_{\text{req}} = 1.2 \cdot W_t = 15.6 \text{ lbf} \]  
...Lift requirement for pull up

\[ C_{L3D} = 0.238 \]  
...3D lift coefficient at 100k

\[ C_{D3D} = 0.028 \]  
...3D drag coefficient at 100k

\[ \text{mass} := \frac{W_t}{g} \]  
...mass of aircraft

\[ n := 1.5 \]  
...Pull up load factor

\[ z_0 := 100000 \text{ ft} \]  
...Drop altitude

\[ v_0 := 0 \cdot \frac{g}{s} \]  
...Starting velocity

\[ L_0 := 0 \]  
...Initial lift

\[ t_0 := 0 \text{ s} \]  
...Initial time

\[ \Delta t := 0.1 \text{ s} \]  
...time increment

\[ i := 1 \]  
...initial index
Glider Fall and Pull Up Cont.

\[ z_{\text{fall}} := \]

for \( i \in 1..200 \)

\[
\begin{align*}
  z_i &\leftarrow z_{i-1} - V_{i-1} \Delta t \\
  D_i &\leftarrow \frac{1}{2} \Phi(z_{i-1}) (V_{i-1})^2 S_{D3D} \\
  V_i &\leftarrow V_{i-1} + \frac{W_t - D_i}{\text{mass}} \Delta t \\
  L_i &\leftarrow \frac{1}{2} \Phi(z_{i-1}) (V_{i-1})^2 S_{L3D} \\
  \text{break if } L_i \geq L_{\text{req}} \quad \text{otherwise} \\
  i &\leftarrow i + 1
\end{align*}
\]

\( x_i \leftarrow 0 \)

\( V_{x_i} \leftarrow 0 \)

\( y_i \leftarrow 0 \)

\( \text{cnt} \leftarrow 0 \)

for \( j \in i..i+500 \)

\[
\begin{align*}
  \gamma_j &\leftarrow 1.571 \quad \text{if } \gamma_{j-1} \geq 1.571 \\
  &\leftarrow \gamma_{j-1} + \frac{g}{V_{j-1}} (n-1) \Delta t \quad \text{otherwise} \\
  z_j &\leftarrow z_{j-1} - V_{j-1} \Delta t \\
  D_j &\leftarrow \frac{1}{2} \Phi(z_{j-1}) (V_{j-1})^2 S_{D3D} \\
  V_j &\leftarrow V_{j-1} + \frac{W_t - D_j}{\text{mass}} \Delta t \cos(\gamma_{j-1}) \\
  V_{x_j} &\leftarrow V_{x_{j-1}} + \frac{V_j - V_{j-1}}{\text{mass}} \Delta t \cos(\gamma_{j-1}) \\
  V_{y_j} &\leftarrow V_{y_{j-1}} + \frac{W_t - D_j}{\text{mass}} \Delta t \sin(\gamma_{j-1}) \\
  x_j &\leftarrow x_{j-1} + V_{x_j} \Delta t \\
  y_j &\leftarrow y_{j-1} + V_{y_j} \Delta t \\
  L_j &\leftarrow \frac{1}{2} \Phi(z_{j-1}) \left[ \left(\frac{V_j}{V_{j-1}}\right)^2 + \left(\frac{V_{x_j}}{V_{x_{j-1}}}\right)^2 \right] S_{L3D} \\
  \text{cnt} &\leftarrow \text{cnt} + 1 \quad \text{if } \gamma_{j-1} \geq 1.571 \\
  &\leftarrow 0 \quad \text{otherwise} \\
  \text{break if cnt} \geq 10 \\
  j &\leftarrow j + 1
\end{align*}
\]

\[ k := 0..679 \]
**Glide**

\[ C_{D0} = 0.024 \quad C_{L\text{star}} = 0.602 \quad k = 0.066 \quad E_{\text{max}} = 12.532 \]

\[ q = \frac{W_{\text{to}}}{S} \sqrt{\frac{k}{C_{D0}}} \quad q = 2.026 \text{psf} \]

The lift coefficient for the smallest glide angle is \( C_{L\text{star}} \)

**Minimum Drag:**

\[ D_{g} = 2W_{\text{to}}\sqrt{k \cdot C_{D0}} \quad D_{g} = 1.037 \text{lbf} \]

**Minimum Glide Angle:**

\[ \gamma_{\text{min}} = \frac{-1}{E_{\text{max}}} \quad \gamma_{\text{min}} = -4.572 \text{deg} \]

**Maximum Range in a glide with min glide angle:**

Specify starting and ending altitudes

\[ z_i := 100000 \text{ft} \quad z_f := 60000 \text{ft} \]

\[ x_{\text{maxg}}(\Delta h) = E_{\text{max}}(\Delta h) \quad x_{\text{maxg}}(40000\text{ft}) = 94.941 \text{mi} \]

**Glide at minimum Sink at a specified altitude**

\[ h_{\text{MS}} := 90000 \text{ft} \quad D_{\text{MinSink}} := \frac{2W_{\text{to}}}{\sqrt{3}E_{\text{max}}} \quad D_{\text{MinSink}} = 1.198 \text{lbf} \]

\[ V_{z\text{MinSink}}(h) = \frac{2W_{\text{to}}}{E_{\text{max}}\rho(h)S} \left( \frac{k}{27C_{D0}} \right)^{\frac{1}{4}} \quad V_{z\text{MinSink}}(h_{\text{MS}}) = 1.171 \times 10^{\frac{3}{8}} \text{min} \quad V_{z\text{MinSink}}(10000\text{ft}) = 177.184 \text{min} \]

**Minimum Glide angle for Minimum Sink**

\[ \gamma_{\text{MinSink}} := \frac{-2}{\sqrt{3}E_{\text{max}}} \quad \gamma_{\text{MinSink}} = -5.279 \text{deg} \]

**Range for Minimum Sink**

\[ x_{\text{MinSink}}(\Delta h) = \frac{\sqrt{3}}{2}E_{\text{max}}(\Delta h) \quad x_{\text{MinSink}}(20000\text{ft}) = 41.111 \text{mi} \]

**Endurance for Minimum Sink**

\[ t_{\text{MinSink}} := \frac{E_{\text{max}}}{P} \left( \frac{27C_{D0}}{k} \right)^{\frac{1}{4}} \sqrt{\frac{1}{8} \left( \frac{1}{2} \frac{1}{2} \right)} \quad t_{\text{MinSink}} = 74.962 \text{min} \]

\[ V_{\text{MS}} := \frac{x_{\text{MinSink}}(3000\text{ft})}{t_{\text{MinSink}}} \quad V_{\text{MS}} = 0.429 \text{knot} \]