Grid fault ride-through capability of voltage-controlled inverters for distributed generation applications

By

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A Dissertation
Submitted to the Faculty of
Mississippi State University
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in Electrical and Computer Engineering
in the Department of Electrical and Computer Engineering

Mississippi State, Mississippi

May 2017
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The increased integration of distributed and renewable energy resources (DERs) has motivated the evolution of new standards in grid interconnection requirements. New standards have the requirement for the DERs to remain connected during the transient grid fault conditions and to offer support to the grid. This requirement is known as the fault ride-through (FRT) capability of the inverter-based DERs and is an increasingly important issue.

This dissertation presents the FRT capability of the DERs that employ a voltage control strategy in their control systems. The voltage control strategy is increasingly replacing the current control strategy in the DERs due to the fact that it provides direct voltage support. However, the voltage control technique limits the ability of direct control over the inverter current. This presents a challenge in addressing the FRT capability where the problem is originally formulated in terms of the current control.
This dissertation develops a solution for the FRT capability of inverters that use a voltage control strategy. The proposed controller enables the inverter to ride through the grid faults and support the grid by injecting a balanced current with completely controlled real and reactive power components. The proposed controller is flexible and can be used in connection with various voltage control strategies. Stability analysis of the proposed control structure is performed based on a new linear time-invariant model developed in this dissertation. This model significantly facilitates the stability and design of such control loops.

Detailed simulation, real-time and experimental results are presented to evaluate the performance of the proposed control strategy in various operating conditions. Desirable transient and steady-state responses of the proposed controller are observed. Furthermore, the newly established German and Danish grid fault ride-through standards are implemented in this research as two application examples and the effectiveness of the dissertation results are illustrated in the context of those two examples.

Key words: fault ride-through, low-voltage ride-through, microgrid, distributed energy resources, voltage sag, renewable energy integration, ancillary services.
DEDICATION

To my Mom Mankumari, Dad Banshiram, and sisters Rashmi, Rasia, and Ranjana.

Thanks for always being there for me.
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor, Dr. Masoud Karimi for his continuous guidance, support, and motivation during my Ph.D study and research. Thanks for directing me to the right path whenever I needed it.

I would like to thank my committee members, Dr. Yong Fu, Dr. Sherif Abdelwahed, and Dr. Heejin Cho for their insightful comments and encouragement.

I would like to thank the Department of Electrical and Computer Engineering, Mississippi State University and its staff for their continued support throughout my graduate study.

I would also like to thank Dr. Sayed Ali Khajehoddin and his research team in University of Alberta, Canada for providing us with their insightful discussions and also the laboratory facilities to carry out the experimental results.

Thanks to all the friends and Nepalese community at Starkville for their continuous encouragement during the challenges of graduate school.

I would also like to thank my parents and sisters for their love, faith, support, and inspiration.
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<tr>
<td>AARC</td>
<td>Average Active-Reactive Control</td>
</tr>
<tr>
<td>APSC</td>
<td>Average Positive Sequence Control</td>
</tr>
<tr>
<td>AUPFC</td>
<td>Average Unity Power Factor Control</td>
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<tr>
<td>BPSC</td>
<td>Balanced Positive Sequence Control</td>
</tr>
<tr>
<td>CC</td>
<td>Current-Controlled</td>
</tr>
<tr>
<td>CPC</td>
<td>Constant Power Control</td>
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<tr>
<td>DERs</td>
<td>Distributed Energy Resources</td>
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<tr>
<td>DGs</td>
<td>Distributed Generators</td>
</tr>
<tr>
<td>DS</td>
<td>Distributed Storage</td>
</tr>
<tr>
<td>EPLL</td>
<td>Enhanced Phase-Locked Loop</td>
</tr>
<tr>
<td>EPS</td>
<td>Electric Power System</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>FRT</td>
<td>Fault Ride-Through</td>
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<tr>
<td>GC</td>
<td>Grid-Connected</td>
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<tr>
<td>IARC</td>
<td>Instantaneous Active Reactive Control</td>
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<td>ICPS</td>
<td>Instantaneously Controlled Positive-Sequence</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>IPSC</td>
<td>Instantaneous Positive-Sequence Control</td>
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<tr>
<td>IUPFC</td>
<td>Instantaneous Unity Power Factor Control</td>
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<tr>
<td>LL</td>
<td>Line-to-Line</td>
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<td>LLG</td>
<td>Line-Line-to-Ground</td>
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<td>LTI</td>
<td>Linear Time-Invariant</td>
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<td>LVRT</td>
<td>Low-Voltage Ride-Through</td>
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<td>MG</td>
<td>Microgrid</td>
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<td>MPPT</td>
<td>Maximum Power Point Tracking</td>
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<td>PCC</td>
<td>Point of Common Coupling</td>
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<td>PDF</td>
<td>Probability Density Function</td>
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<td>PI</td>
<td>Proportional-Integrator</td>
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<td>PLL</td>
<td>Phase-Locked Loop</td>
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<td>PNSC</td>
<td>Positive-Negative-Sequence Compensation</td>
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<td>pu</td>
<td>per-unit</td>
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<td>PV</td>
<td>Photovoltaic</td>
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<td>PWM</td>
<td>Pulse Width Modulation</td>
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<td>RMS</td>
<td>Root-Mean-Square</td>
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<tr>
<td>RTDS</td>
<td>Real-Time Digital Simulator</td>
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<td>SA</td>
<td>Standalone</td>
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<td>--------------------------------</td>
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<tr>
<td>SCC</td>
<td>Short Circuit Current</td>
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<td>SLG</td>
<td>Single-Line-to-Ground</td>
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<td>UC</td>
<td>Universal Controller</td>
</tr>
<tr>
<td>VC</td>
<td>Voltage-Controlled</td>
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<tr>
<td>VSC</td>
<td>Voltage Source Converter</td>
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<td>VSG</td>
<td>Virtual Synchronous Generator</td>
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CHAPTER 1

INTRODUCTION

1.1 Evolutions in Modern Electric Power System

The Electric Power System (EPS) consists of the generation, transmission, and distribution of the electric power to the electrical loads. It has a vertical structure where the power flows from the generation to the distribution through the transmission network. The conventional generation units are the large power plants which are mainly based on fossil fuels, nuclear energy, and large hydro power plants. The power plants based on coal or fossil fuels are one of the major sources of air pollution. The greenhouse gases produced by these power generation units are believed to be a major cause of global warming and climate change. Nuclear power plants and hydro power plants have their own environmental issues as well. The renewable energy technologies such as the wind and solar energy technologies are being developed rapidly as an alternative source of energy to deliver a cleaner energy in a more sustainable way.

The electricity demand is ever-increasing and the capacity of generation, transmission and distribution networks to meet this demand is a major constraint. The deregulation of the power industry allowed integration of distributed and renewable energy generators at dispersed locations. Dispersed or Distributed Generators (DGs) are small generation systems connected to the distribution (low-voltage) or sub-transmission system
Examples of DG systems are solar photovoltaic, solar thermal, and wind (among renewable-based sources) and micro-turbines, and diesel generators (among nonrenewable-based sources). The Distributed Storage (DS) devices can also be included among such generators. The benefits of DG systems are reduction in transmission losses; reducing required level of transmission system expansions; recovering the heat loss (by using combined heat power systems); deregulated power market; and possibility of autonomous operation of a section of distribution system called an islanded microgrid.

Despite their clear advantages, the DG systems have several challenges associated with them. These challenges are related to the high level of penetration of those systems. They are mainly listed as follows.

1. Bidirectional power flow requires modification of the EPS protection equipment.
2. Voltage profile changes may require modification of voltage regulators’ and other compensating devices’ settings.
3. Grid stability problems due to variable nature of renewable sources.
4. Islanding prevention (when the local EPS is unavailable, the DGs should not energize it).
5. Coordination and control of high number of DGs.
6. Grid-scale storage systems may be required.

Microgrid (MG) concept has been proposed to address some of these challenges. A microgrid is a cluster of distributed energy resources (DERs) (including distributed generator, distributed storage, and distributed controllable loads) and passive loads that work
together as a single controllable unit. The concept of the MG is essential for the realization of the active distribution network where control is distributed and power flow is bidirectional. The MG is connected to the grid using a main switch device and it can operate in grid-connected (GC) as well as in islanded or standalone (SA) modes. Properties and advantages of the MG may be described as follows.

1. Operation in grid-connected and islanded modes

2. When connected to the grid, the MG performs as a controllable entity with controlled interaction with the rest of the grid. That interaction may be characterized as follows.
   - Controlled real and reactive power exchange
   - Harmonic filtering ability
   - Fault ride through capability
   - Grid stability support and improvement
   - Power quality improvement; smooth and balanced current

3. When isolated from the grid, the MG supplies power to its own loads. Its operation is characterized by following aspects.
   - Voltage quality and stability
   - Power management and power sharing

4. The MG must make seamless transition from GC to SA and vice versa.

5. Smart grid functionalities can be realized using MG concept.
The ability of a MG to operate in both GC and SA modes is a key advantage to increase reliability and resilience of the EPS. The IEEE standard on Interconnecting Distributed Resources with Electric Power Systems, IEEE Standard 1547 [1], requires islanding of the DERs in cases where the grid is not in a normal condition. The DER must disconnect from the grid within 2 s and avoid formation of an unintentional island which means a portion of the local EPS is powered by the MG. Therefore, the MG must be equipped with an islanding detection algorithm to separate from the grid within this time frame. Further increases in the penetration level of DERs proved that this passive operation is not desirable. And the standards are modified to require the DERs to remain connected to the grid and provide grid support during grid transients as is summarized in the following section.

1.2 Grid Codes and Fault Ride-Through (FRT) Strategy

The Fault Ride-Through (FRT) or Low-Voltage Ride-Through (LVRT) capability describes the ability of the DER to remain connected and supportive during transient grid faults. The DERs are required to disconnect from the grid during the grid faults according to the current standards of practice [1]. In recent years, the penetration of renewable energy sources such as wind and photovoltaics has significantly increased in the present grid [4, 47]. Wind farms and PV plants of large capacity are being integrated to the grid to meet the ever increasing energy demands in a sustainable way. Due to the high share of power delivery from those DERs, their sudden disconnection from the grid during the fault situations further increases the stress on the grid and may lead to instabilities. So the
present grid codes are continually being modified with the requirement for the DERs to remain connected to the grid and to support the grid during the transient faults.

The grid codes of various different countries have defined the boundaries of the FRT capability of the DERs in slightly different ways [2, 3, 34, 51]. The LVRT time-voltage profiles for several different countries are shown in Figure 1.1. This graph shows how long the DG should remain connected for every level of grid voltage drop. Figure 1.2 shows the LVRT time-voltage profile for PV power plants according to Danish Grid Code. According to the grid code, the PV power plants should remain connected to the grid for a voltage dip to 10% of the nominal voltage at the PCC for a period of 250 ms. The LVRT time-voltage profile defines three specific modes of operation of the PV plant during symmetrical and asymmetrical faults as shown in Figure 1.2.

- Area A: The PV power plant should operate normally in this region.
- Area B: The PV power plant should remain connected to the grid and support the grid by reactive current injection.
- Area C: The PV power plants are allowed to disconnect from the grid.

During the short-term voltage dips in the system, the DERs should remain connected to the grid and supports the grid, e.g. through reactive power injection. The requirement of reactive current injection is different for different countries. For example, the German grid code [7] has requirement of reactive current injection during the grid faults as defined in Figure 1.3 for the wind power plants. For the German grid code the requirements are summarized as:
Figure 1.1
LVRT time-voltage profile for various countries

Figure 1.2
LVRT time-voltage profile for PV power plant according to Danish Grid Code [2]
Figure 1.3

Reactive current requirement during grid faults according to German Standard

Figure 1.4

Reactive current requirement during grid faults according to Danish Standard
• When the voltage moves away from its nominal value, the reactive current has
to be injected to the grid as shown in Figure 1.3. A deadband of 10% of voltage
deviation is used.

• The slope of the reactive current injection can be changed between 2 to 10.

• For asymmetrical faults, the maximum limit of the reactive current injection is
limited to 40% of the rated value.

• The active power can be reduced to allow the supply of the reactive current as
required.

• After the fault clearance, the active power injection must be restored to original
value with the gradient of at least 20%/s.

Similarly, Danish grid code has the reactive current injection requirement for PV power
plants as defined by Figure 1.4. These requirements are explained as:

• During symmetrical or asymmetrical grid faults, the PV power plants must inject
the positive sequence reactive current.

• The control must follow Figure 1.4 while injecting positive sequence reactive cur-
rent with a tolerance of ±20%.

• Delivery of the reactive power is kept in the first priority during the voltage sag.

• The active power supply must be maintained during the voltage sag but it can be
reduced to meet the requirement of reactive power injection.

The grid faults usually appear as an unbalanced voltage sag at the point of connection of
the DG to the grid. This will immediately cause unbalanced and/or excessively increased
DG currents. The first objective of a FRT control scheme is to ensure that the currents remain within acceptable limits. Moreover, as far as the operation of the DG during the fault intervals is concerned, in addition to having controlled amount of real and reactive powers, the common (but not unique) strategy is that the DG should supply balanced positive sequence current to the grid [49].

To summarize, a DER with a desired control system is responsible for the following tasks as far as the LVRT requirements are concerned.

- Detect the fault instant and activate the control system to implement the LVRT requirements by a given standard.
- Prevent transient over-currents.
- Control real and reactive power exchange according to the standard.
- Supply controlled balanced positive sequence current.

The distributed resources are generally interfaced to the grid using Voltage Source Converters (VSCs) [8, 9, 28]. The grid-connected VSCs are commonly controlled using current control techniques such as the popular vector control in synchronous reference frame or in stationary domain [9]. There are also different current control modifications for the inverters to operate under unbalanced grid voltage and they can be used in principle to address the FRT capability [11, 19, 32, 37, 43, 45, 49].

For emerging MG applications, however, the voltage control techniques have been drawing more attention in recent years [20, 38, 42, 53]. Reasons lie in the fact that the MGs need to operate in islanded as well as grid-connected mode and the voltage control techniques lend themselves well for such operational requirements. Despite their significant
advantages, the voltage control techniques pose difficulties for FRT applications where the control problem is directly towards controlling the current. Due to lack of direct control over the current, the FRT control objectives stated above cannot be addressed directly by a voltage control method. Hence, methods should be developed to achieve those objectives indirectly through the voltage control loop. To the best of our knowledge, there appears to be no major work reported on VSCs operated with voltage control techniques that can satisfy the FRT capability and address the aforementioned objectives.

1.3 Proposed Research Objectives

Accordingly, the objectives of this research is to develop a control system that can address the LVRT requirements in a voltage-controlled DER. Meanwhile, since there are multiple voltage control techniques in the literature, we plan to develop our controller in such a way that it can be applied to all of them without requiring changes in their internal structures. Thus, the objectives of this research are summarized as follows.

- Identify a representative voltage control technique to address the MG control requirements.
- Develop a control structure that can be employed as an addendum to the DER control system to address the control objectives for the FRT capability stated in Section 1.2.
- Identify or develop a fault detection method that can be used in conjunction with the LVRT controller developments.
- Validate the developments using computer and real-time simulations.
• The developed LVRT controller must be applicable to other voltage control tech-
niques.

1.4 Contributions

This work develops, for the first time, a control strategy to address the LVRT require-
ments in the voltage-controlled DERs. Specific contributions of the work are summarized
as follows.

1. Comprehensive literature review on existing LVRT methods and controllers for
current-controlled and voltage-controlled DERs.

2. Development of a new control structure to address the LVRT requirements in a
voltage-controlled DER. The proposed controller can be flexibly added to any
voltage-controlled DER without changing the structure of the controller.

3. Complete stability study of the proposed controller in conjunction with an exist-
ing voltage control technique.

4. Derivation and validation of a new modeling technique for the stability analysis
of the voltage controlled DERs.

5. Identification and adjustment of a fault detection method to be used with the
proposed LVRT controller.

6. Extensive off-line as well as real-time simulation studies of the proposed con-
trolled in as many operating conditions as possible.
CHAPTER 2

LITERATURE REVIEW

In countries with the large penetration of distributed and renewable energy resources (DERs), the grid codes regarding the interconnection of these DERs are being modified and updated with the new requirements. The new grid codes have the requirement for these DERs to remain connected to the grid during grid faults and provide support to the grid. This support is mainly characterized by a controlled reactive power injection. Therefore, the DERs should be equipped by appropriate control algorithms in order to enable them to meet these requirements called Low-Voltage Ride-Through (LVRT) or Fault Ride-Through (FRT) requirements. The major problems during grid faults are over-current, unbalanced current, and large uncontrolled oscillations in both instantaneous active and reactive powers.

Various operating strategies for DERs during grid faults are discussed in [43] where the focus is on real power control during the fault. The Instantaneous Active Reactive Control (IARC) generates a current vector which is instantaneously parallel to the grid voltage [43] to ensure that smooth power is delivered but it will generate a distorted and unbalanced current. The Instantaneously Controlled Positive-Sequence (ICPS) method generates a current vector which is instantaneously parallel to the grid voltage positive
sequence component [43] to reduce the current distortion as compared to IARC but the reactive power becomes oscillatory at double frequency. The Positive-Negative-Sequence Compensation (PNSC) method will inject an additional negative sequence current (making the total current unbalanced but sinusoidal) to cancel the double frequency power ripples in active power while large oscillations are present in reactive power [43]. In the Average Active-Reactive Control (AARC) method [43], the current vector is selected monotonously proportional to the grid voltage and thus becomes unbalanced during the grid faults but this will remove the harmonics from the current in IARC method. The reactive power oscillation is also removed but the active power has large double frequency oscillations. Finally, the Balanced Positive Sequence Control (BPSC) method [43] feeds a balanced sinusoidal current proportional to the grid voltage positive sequence component. Thus, it will cause double frequency oscillations in both active and reactive powers during the grid faults.

In [11], a flexible current reference calculation strategy with capability to limit peak current and flexibly determine the positive and negative sequence powers is proposed for the operation of the DERs during grid faults. This method is a modification of PNSC with the ability to limit the current through adjusting the real power or the reactive power.

In [5], three different strategies are discussed to generate the current reference for the operation of the converters during grid faults. The first strategy is similar to BPSC that injects balanced positive sequence current to the grid. Second strategy is similar to PNSC where real power oscillations are removed and current reference is calculated using real and reactive power references. The reactive power oscillations are present in this strategy.
In third strategy, the current references are calculated such that the oscillating real power at the internal converter terminals are cancelled. In this way, no oscillating active power flows to/from the converter DC side that reduces disturbances and oscillations at the DC bus of the inverter.

Various control strategies in the context of PV inverters are proposed to achieve LVRT requirements [13, 32, 36, 48]. In [13], different current reference generating strategies are discussed. The first strategy called Instantaneous Unity Power Factor Control (IUPFC), which is same as the IARC strategy, generates and injects a current that exactly follows the voltage. Next strategy is the Average Unity Power Factor Control (AUPFC), which is the same as AARC strategy, where the current harmonics are eliminated. Third strategy is the Instantaneous Positive-Sequence Control (IPSC), same as the ICPS strategy, where the current follows the positive sequence voltage. Next strategy is the Average Positive Sequence Control (APSC) same as the BPSC. Finally, the last strategy is the Positive Negative Sequence Compensation Control (PNSCC), same as the PNSC, which generates and injects smooth real power and eliminates current harmonics. In [32], a current limiting feature is added to the control strategies discussed in [13]. In [48], a flexible current controller is proposed which can operate in three different modes based on the severity of the voltage sag. First, a Maximum Power Point Tracking (MPPT) mode is adopted if the grid voltage is between 0.9 pu to 0.5 pu. In this mode, the power reference is obtained from the MPPT algorithm if it is below the maximum allowable value indicated by the LVRT requirements and the current margins of the PV inverter. If the power output of PV inverter exceeds that maximum allowable value, the inverter is operated in Constant Power Control (CPC)
mode where the MPPT algorithm is disabled and the power reference is set to this maximum allowable value. If the grid voltage is below 0.5 pu, the Short Circuit Current (SCC) mode is activated where the inverter will inject full reactive power with no real power. An LVRT control strategy for single phase PV inverters is proposed in [36] with the adjustable tradeoff between the power oscillations and the current harmonics. The real and reactive power references are set arbitrarily according to the LVRT requirements.

A control strategy that utilizes not only the positive and negative sequence currents but also the zero-sequence current is proposed in [30]. The three-phase four-wire inverter with the zero-sequence current path is used here. The access to the zero-sequence current gives extra control freedom to fulfill additional control requirements. It is shown in this paper that with the zero-sequence current control, the active (or reactive) power oscillations can be eliminated along with having a balanced current injection during unbalanced grid-voltage sags; something that is not possible in a three-wire inverter.

Different voltage support control strategies for the DERs during grid faults have been introduced in [12, 31, 33, 35]. In [12], a voltage support strategy based on the type of the voltage sag is proposed. For the symmetrical voltage sags, the voltage support strategy supplies positive sequence reactive current to raise the voltages in all three phases. For the voltage sags in one phase or two phases, the voltage support strategy supplies positive and negative sequence reactive currents to try to get equal voltages on all phases. In addition, this strategy also has current limiting function to avoid overcurrent problem and the generated current is sinusoidal (but unbalanced). In [12], the reactive power reference required to generate the positive and negative sequence reactive current references is calcu-
lated offline. In [31], a voltage control strategy is developed based on the voltage support strategy of [12] that generates online values for the reactive power reference and the control parameter based on the type of the voltage sag. In [33], current references for each phase current is generated individually to supply the reactive power in each phase based on the voltage drop on that phase. This strategy has an advantage of avoiding overvoltage in healthy phases in case of unbalanced voltage sag.

Two control scenarios based on the control of amplitude and phase angle of the negative sequence current are presented in [35] for unbalanced grid voltage compensation. First method attempts to reduce the active power oscillations by controlling the negative and positive sequence real and reactive power injection to the grid. Second method reduces the negative sequence voltage generated due to grid unbalance by generating the negative sequence current in-phase with the negative sequence current of the grid.

All the LVRT control strategies reviewed above are based on generating the reference current that can address the requirements. The LVRT requirements are not unanimously defined as far as the waveform of the current is concerned. However, limiting the peak value of the injected current and control of the real and reactive powers during the grid fault are necessary in all of them. Once the reference current is generated, the LVRT control is readily implemented if the DER is adopting a Current Control (CC) loop. Current limiting is also directly achieved in CC-DERs. The CC strategy, however, is shown to fail to address operation of the DER in islanded and in weak grid conditions. Thus, the voltage-controlled DERs are increasingly being introduced for such conditions.
The challenge is to implement the LVRT control for Voltage-Controlled (VC) DERs where a current control loop is not available. The voltage controlled inverters are suitable for microgrid applications where operation of the DERs in both stand-alone and grid-connected modes is desired. Different direct voltage control strategies adopted so far can be classified into those which are based on direct droop principles [18, 50, 52], virtual synchronous generators [15, 46], synchronverters [53, 54], and universal controllers [24, 26]. The fault ride-through capability of these voltage control strategies along with the complete stability analysis have not been fully studied yet.

The performance of the Virtual Synchronous Generator (VSG) for voltage sag ride-through is studied in [6, 44]. The VSG is an inverter control algorithm that imitates the operation of synchronous generator based on the swing equation. The virtual inertia is realized by the energy storage device. Conventional VSG controller has overcurrent problem during unbalanced grid-voltage conditions. Three modifications are introduced to address this overcurrent issue. First, a voltage amplitude control is implemented which calculates the RMS value of the grid voltage and uses it to limit the inverter output voltage [6]. Second, the output power control adjusts the power reference proportional to the square of the RMS voltage calculated by voltage amplitude controller. Third, a virtual inertia control is added that provides additional damping to reduce power oscillations after the fault is cleared. A similar method is presented in [25] to limit the real and reactive power references against the grid voltage sags. This method and also the method of [6] will limit the current in single-phase grids and three-phase balanced faults and cannot work properly in unbalanced faults.
The overcurrent and overload protection strategy for voltage controlled DERs of an islanded microgrid is presented on [17]. The overcurrent protection strategy has fault detection unit, current limiting unit, and a unit to detect fault clearance. When a fault is detected, the output voltage from the controller is ignored and the terminal voltage is calculated based on the grid voltage phasor, the impedance between the inverter terminal and the Point of Common Coupling (PCC), and the prefault output current. This modified voltage limits the output current of the inverter. After the fault clearance, the voltage controller is activated. The overload protection strategy limits the power output during the fault. The difference between the inverter terminal voltage and the PCC voltage is used to detect overload condition. At overload condition, the output voltage set by the controller is ignored and the modified voltage value is set based on PCC voltage phasor and the maximum allowable voltage difference that corresponds to the maximum power output. This strategy limits the output power of the inverter.

The literature review illustrates that the LVRT is not addressed for VC-DERs in any detailed and comprehensive manner close to what is done for CC-DERs. A few publications, described above, discuss current limiting and overload protection of the VC-DERs in a limited way. This dissertation proposes to address the LVRT operation of VC-DERs. We propose to add an auxiliary controller that can be added to any VC-DER to fulfill the LVRT requirements. While the proposed controller addresses the LVRT requirements (as specified by dominant standards), it does not require a change inside the original voltage control scheme. The proposed controller works as an add-on with any voltage control algorithm. The complete stability analysis of the proposed controller is also performed.
CHAPTER 3
PROPOSED CONTROLLER

An auxiliary voltage control structure is developed to address the LVRT control challenges for the voltage controlled DERs in this work. This auxiliary controller can work with any voltage controller without altering its structure and stability. The voltage control method called the Universal Controller (UC) of [24, 26, 27] is adopted as a representative voltage control method for the study purpose in this research. Therefore, this chapter first reviews the adopted control system before discussing the proposed Fault Ride-Through (FRT) solution.

3.1 Review of Universal Control Structure

Consider the simplified block diagram of a three-phase grid connected inverter as shown in Figure 3.1. If $L$ is the interfacing filter inductance, the system equation is

$$v_{\text{inv}}(t) = L\frac{di(t)}{dt} + v_g(t) \quad (3.1)$$

where $v_{\text{inv}}(t)$ is the inverter voltage, $v_g(t)$ is the grid voltage and $i(t)$ is the inverter current. The virtual inverter voltage is defined as $v(t) = v_{\text{inv}}(t) + Ri(t)$ where $R$ is a constant
playing the role of a virtual resistor. The structure of UC designed in $\alpha\beta$ reference frame is shown in Figure 3.2 where $T(\theta)$ is a rotation matrix defined as

\[
T(\theta) = \begin{pmatrix}
\sin \theta & -\cos \theta \\
\cos \theta & \sin \theta
\end{pmatrix} = \frac{1}{Z} \begin{pmatrix} X & -R \\ R & X \end{pmatrix}
\]

(3.2)

and $\theta$ is defined by $\vec{Z} = R + jX = Z e^{j\theta}$, $X = L\omega$, and $\omega$ is the system’s frequency in rad/s [26].

In order to explain the derivation stage of the UC, let us define the virtual inverter voltage phasor as $\vec{V} = V \angle \delta$ and the output voltage phasor as $\vec{V}_{out} = V_{out}$ where $V$ and $V_{out}$ are the peak values; and $\delta$ is the phase angle. The virtual real and reactive powers at
the $v$ terminals are obtained using $\vec{S} = \vec{V} \vec{I}^* = P + jQ$ that results in the following two expressions

$$P = \frac{3V}{2Z}[V \cos \theta - V_{\text{out}} \cos(\delta + \theta)]$$

$$Q = \frac{3V}{2Z}[V \sin \theta - V_{\text{out}} \sin(\delta + \theta)].$$

Then, the transformed virtual powers are

$$\begin{pmatrix} P' \\ Q' \end{pmatrix} = T(\theta) \begin{pmatrix} P \\ Q \end{pmatrix} = \frac{3V}{2Z} \begin{pmatrix} \sin \delta \\ \frac{V - V_{\text{out}} \cos \delta}{V_{\text{out}}} \end{pmatrix}$$

$$\begin{pmatrix} P' \\ Q' \end{pmatrix} = T(\theta) \begin{pmatrix} P \\ Q \end{pmatrix} = \frac{3V}{2Z} \begin{pmatrix} \sin \delta \\ \frac{V - V_{\text{out}} \cos \delta}{V_{\text{out}}} \end{pmatrix}.$$ 

The transformed virtual powers have the property that real and reactive powers are dominantly controlled with the angle, $\delta$, and the magnitude, $V$, respectively. Therefore, the following approximations are considered.

$$\frac{\partial P'}{\partial V} \approx 0, \quad \frac{\partial Q'}{\partial \delta} \approx 0.$$ 

If $P'_{\text{ref}}$ and $Q'_{\text{ref}}$ are the reference values for the transformed powers, then the cost function is selected as

$$J = \frac{1}{2}(P' - P'_{\text{ref}})^2 + \frac{1}{2}(Q' - Q'_{\text{ref}})^2.$$ 

The gradient descent method is used to minimize the cost function. The gradient descent method is expressed by

$$\frac{d}{dt} \Theta = -\mu \frac{\partial}{\partial \Theta} J(\Theta)$$

where $J(\Theta)$ is the cost function and $\mu$ is a constant. This implies that if $P'_{\text{ref}}$ and $Q'_{\text{ref}}$ are the reference values for the transformed powers, then the following equations can be derived to track those references [24, 26, 27].

$$\frac{d}{dt} V(t) = k_q(Q'_{\text{ref}} - Q')$$

$$\frac{d}{dt} \delta(t) = k_p(P'_{\text{ref}} - P').$$
In Equation (3.8), $k_p$ and $k_q$ are some positive constants.

In a balanced three-phase system, the virtual real and reactive powers $P$ and $Q$ are equal to their instantaneous values and given by

$$
P = p(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) = \frac{3}{2}[v_\alpha(t)i_\alpha(t) + v_\beta(t)i_\beta(t)]$$

$$
Q = q(t) = v'_a(t)i_a(t) + v'_b(t)i_b(t) + v'_c(t)i_c(t) = \frac{3}{2}[v_\beta(t)i_\alpha(t) - v_\alpha(t)i_\beta(t)]
$$

where $v'_a(t)$ is 90-degree delayed version of $v_a(t)$. The $\alpha\beta$ variables are obtained from the well-known Clarke transformation ($3 \rightarrow 2$ transformation):

$$
\begin{bmatrix}
  v_\alpha \\
  v_\beta
\end{bmatrix} =
\begin{bmatrix}
  \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
  0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3}
\end{bmatrix}
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c
\end{bmatrix}.
$$

In a balanced three-phase system, the abc voltages can be written as

$$
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c
\end{bmatrix} =
\begin{bmatrix}
  V\cos(\omega t) \\
  V\cos(\omega t - 120^\circ) \\
  V\cos(\omega t + 120^\circ)
\end{bmatrix}.
$$

In this case, the $\alpha\beta$ signals may be expressed as

$$
\begin{bmatrix}
  v_\alpha \\
  v_\beta
\end{bmatrix} =
\begin{bmatrix}
  V\cos(\omega t) \\
  V\sin(\omega t)
\end{bmatrix}.
$$

Then Equation (3.8) is expressed as

$$
\frac{d}{dt}\begin{bmatrix}
  \delta(t) \\
  V_i(t)
\end{bmatrix} =
\begin{bmatrix}
  k_p & 0 \\
  0 & k_q
\end{bmatrix}Y(\theta)\begin{bmatrix}
  P_{\text{ref}} - p(t) \\
  Q_{\text{ref}} - q(t)
\end{bmatrix}.
$$

Figure 3.2 shows the realization of Equation (3.13). An addition is an integrator to adjust the inverter frequency due to the grid frequency changes inspired from the PLL.
theory [22]. This is shown by the integrator with gain $k_\omega$ in Figure 3.2. The constant $\omega_n$ shows the nominal grid frequency in radians per second.

### 3.2 Formulation of FRT Objectives

The asymmetrical grid faults generally cause an unbalanced voltage drop at the Point of Common Coupling (PCC) between inverter and the grid. For a common 3-phase 3-wire inverter system, there is no flow of zero-sequence current. Therefore, the phase voltages under unbalance condition can be represented as $v = v^+ + v^-$ where $v^+$ is the positive sequence and $v^-$ is the negative sequence component of the voltage. Under unbalance grid conditions, the three phase instantaneous active power $p$ and instantaneous reactive power $q$ can be represented as the sum of an average power and the oscillating (or pulsating) powers as shown in

$$p = P + \tilde{p}$$

$$q = Q + \tilde{q}$$

where $P$ and $Q$ are average active and reactive powers and $\tilde{p}$ and $\tilde{q}$ are oscillating (or pulsating) terms. These powers are given by

$$P = \frac{3}{2}(v_\alpha^+ i_\alpha^+ + v_\beta^+ i_\beta^+ + v_\alpha^- i_\alpha^- + v_\beta^- i_\beta^-)$$

$$Q = \frac{3}{2}(v_\alpha^+ i_\alpha^- - v_\alpha^- i_\alpha^+ + v_\beta^- i_\beta^- - v_\beta^- i_\beta^+)$$

$$\tilde{p} = \frac{3}{2}(v_\alpha^+ i_\alpha^+ + v_\beta^- i_\beta^- + v_\alpha^- i_\alpha^- + v_\beta^- i_\beta^-)$$

$$\tilde{q} = \frac{3}{2}(v_\alpha^- i_\alpha^+ - v_\alpha^- i_\alpha^+ + v_\beta^- i_\beta^- - v_\beta^- i_\beta^-).$$
The pulsating powers are sinusoidal at double the frequency of the grid. The latter equations can be represented in a matrix form as

\[
\begin{bmatrix}
    P \\
    Q \\
    \bar{p} \\
    \bar{q}
\end{bmatrix} = \frac{3}{2} \begin{bmatrix}
    v_{\alpha}^+ & v_{\beta}^+ & v_{\alpha}^- & v_{\beta}^- \\
    v_{\beta}^+ & -v_{\alpha}^+ & v_{\beta}^- & -v_{\alpha}^- \\
    v_{\alpha}^- & v_{\beta}^- & v_{\alpha}^+ & v_{\beta}^+ \\
    v_{\beta}^- & -v_{\alpha}^- & v_{\beta}^+ & -v_{\alpha}^+
\end{bmatrix} \begin{bmatrix}
    i_{\alpha}^+ \\
    i_{\beta}^+ \\
    i_{\alpha}^- \\
    i_{\beta}^-
\end{bmatrix}
\]

(3.19)

where \(v_{\alpha}^+, v_{\beta}^+, i_{\alpha}^+, i_{\beta}^+\) are the positive sequence voltage and current and \(v_{\alpha}^-, v_{\beta}^-, i_{\alpha}^-, i_{\beta}^-\) are the negative sequence voltage and current in the \(\alpha\beta\) reference frame. The positive and negative sequence average active and reactive powers satisfy \(P^+ + P^- = \tilde{P}\) and \(Q^+ + Q^- = \tilde{Q}\). And they are given by

\[
P^+ = \frac{3}{2} (v_{\alpha}^+ i_{\alpha}^+ + v_{\beta}^+ i_{\beta}^+) \quad (3.20)
\]

\[
P^- = \frac{3}{2} (v_{\alpha}^- i_{\alpha}^- + v_{\beta}^- i_{\beta}^-) \quad (3.21)
\]

\[
Q^+ = \frac{3}{2} (v_{\beta}^+ i_{\alpha}^+ - v_{\alpha}^+ i_{\beta}^+) \quad (3.22)
\]

\[
Q^- = \frac{3}{2} (v_{\beta}^- i_{\alpha}^- - v_{\alpha}^- i_{\beta}^-). \quad (3.23)
\]

From Equation (3.19), for a given set of active and reactive reference powers \(P_{\text{ref}}\) and \(Q_{\text{ref}}\), the reference current can be calculated as

\[
\begin{bmatrix}
    i_{\alpha, \text{ref}}^+ \\
    i_{\beta, \text{ref}}^+ \\
    i_{\alpha, \text{ref}}^- \\
    i_{\beta, \text{ref}}^-
\end{bmatrix} = \frac{2}{3} [U]^{-1} \begin{bmatrix}
    P_{\text{ref}} \\
    Q_{\text{ref}} \\
    \tilde{p} \\
    \tilde{q}
\end{bmatrix}
\]

(3.24)
where

\[
U^{-1} = \frac{1}{\det(U)} \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} \\
C_{12} & -C_{11} & -C_{14} & C_{13} \\
C_{13} & C_{14} & C_{11} & C_{12} \\
-C_{13} & C_{12} & -C_{11} & C_{14}
\end{bmatrix}
\]

\[
\det(U) = \{(v_{\alpha}^+ - v_{\alpha}^-)^2 + (v_{\beta}^+ - v_{\beta}^-)^2\}\{(v_{\alpha}^+ + v_{\alpha}^-)^2 + (v_{\beta}^+ + v_{\beta}^-)^2\}
\]

\[
C_{11} = v_{\alpha}^+ v_{\alpha}^- + v_{\beta}^+ v_{\beta}^- - v_{\alpha}^2 v_{\alpha}^- + v_{\beta}^2 v_{\beta}^- - 2v_{\alpha}^- v_{\alpha}^+ v_{\beta}^-,
\]

\[
C_{12} = v_{\beta}^+ v_{\alpha}^2 v_{\alpha}^- + v_{\alpha}^2 v_{\beta}^+ v_{\beta}^- - v_{\beta}^2 v_{\beta}^- - 2v_{\alpha}^+ v_{\alpha}^- v_{\beta}^-,
\]

\[
C_{13} = v_{\alpha}^- v_{\alpha}^3 + v_{\beta}^- v_{\beta}^3 + v_{\alpha}^2 v_{\alpha}^- v_{\alpha}^2 + v_{\beta}^2 v_{\beta}^- v_{\beta}^2 - 2v_{\alpha}^+ v_{\alpha}^- v_{\beta}^- v_{\beta}^+,
\]

\[
C_{14} = v_{\beta}^- v_{\alpha}^2 v_{\alpha}^- v_{\alpha}^2 - v_{\beta}^2 v_{\beta}^- v_{\beta}^2 + v_{\alpha}^- v_{\alpha}^2 v_{\alpha}^- v_{\alpha}^2 - 2v_{\alpha}^+ v_{\alpha}^- v_{\beta}^- v_{\beta}^+.
\]

Equation (3.24) summarizes the relationship between the current, voltage, and power components of a three-phase system in the general case where the voltage is unbalanced. In grid-connected applications, since the voltage is available, the currents can be calculated based on desired values of the powers. For current-controlled inverters, these calculated currents are considered as the reference values forwarded to a feedback control loop to force the inverter to generate those currents. Several existing approaches for current-controlled inverters are briefly discussed in the following section.

### 3.3 Existing LVRT Strategies for Current-Controlled DERs

Development of the LVRT control strategies for the current-controlled DERs first requires the reference current calculation. Various reference current calculation strategies [13, 43] are discussed here.
Let \( i_{ref} = i^+_{ref} + i^-_{ref} \) be the reference current and \( P_{ref} \) and \( Q_{ref} \) the reference powers. The current references are calculated assuming \( Q_{ref} = 0 \).

In one strategy, during unbalanced voltage sag, a reference current is calculated so as to inject the real and reactive powers without oscillations (or pulsations) that is \( \tilde{p} = \tilde{q} = 0 \). In this case, the reference currents may be calculated and expressed as

\[
\begin{align*}
    i^+_{ref} &= \frac{P_{ref}v^+}{|v^+|^2 + 2v^+v^- + |v^-|^2}, \\
    i^-_{ref} &= \frac{P_{ref}v^-}{|v^+|^2 + 2v^+v^- + |v^-|^2}.
\end{align*}
\] (3.26)

However, the current generated with this reference calculation strategy is highly distorted and unbalanced.

Next strategy calculates the current reference as

\[
\begin{align*}
    i^+_{ref} &= \frac{P_{ref}v^+}{|v^+|^2 + 2v^+v^-}, \\
    i^-_{ref} &= 0.
\end{align*}
\] (3.27)

This strategy reduces the current distortion as compared to Equation (3.26) but the double frequency oscillation in the reactive power is present.

In next strategy negative sequence current is injected to remove the active power oscillation. The reference currents are calculated as

\[
\begin{align*}
    i^+_{ref} &= \frac{P_{ref}v^+}{|v^+|^2 - |v^-|^2}, \\
    i^-_{ref} &= \frac{-P_{ref}v^-}{|v^+|^2 - |v^-|^2}.
\end{align*}
\] (3.28)

In this case, the current injected is sinusoidal but unbalanced. Also, the double frequency oscillation is present in the reactive power.

If the current references are calculated as

\[
\begin{align*}
    i^+_{ref} &= \frac{P_{ref}v^+}{|v^+|^2 + |v^-|^2}, \\
    i^-_{ref} &= \frac{P_{ref}v^-}{|v^+|^2 + |v^-|^2}.
\end{align*}
\] (3.29)
the reactive power has no double frequency oscillation but the real power has double frequency oscillation. In this strategy the current is sinusoidal but unbalanced.

The strategy to inject the balanced positive sequence current to the grid uses the current references calculated as

\[ \hat{i}_{\text{ref}}^+ = \frac{P_{\text{ref}} v^+}{|v^+|^2}, \quad \hat{i}_{\text{ref}}^- = 0. \]  \hspace{1cm} (3.30)

The current is sinusoidal in this case but both the real and reactive powers have double frequency oscillations. In this scenario, if \( Q_{\text{ref}} \) is nonzero, the reference currents are calculated as

\[ \hat{i}_{\text{ref}}^+ = \frac{P_{\text{ref}} v^+}{|v^+|^2}, \quad \hat{i}_{\text{ref}}^* = \frac{Q_{\text{ref}} v^*}{|v^+|^2} \]  \hspace{1cm} (3.31)

where \( v^* \) is the 90° delayed version of the original positive sequence grid voltage \( v^+ \), \( \hat{i}_{\text{Pref}}^+ \) is the real component of the positive sequence current, and \( \hat{i}_{\text{Qref}}^* \) is the reactive component of the positive sequence current and \( \hat{i}_{\text{ref}}^+ = \hat{i}_{\text{Pref}}^+ + \hat{i}_{\text{Qref}}^* \).

Most of the LVRT control strategies for current-controlled DERs use one of the current references given by Equations (3.26)-(3.30) with or without slight modifications. The references are calculated in \( abc \) reference frame, or \( \alpha \beta \) reference frame, or \( dq \) reference frame. The PR or PI controllers are used for both positive and negative sequence current control loops to ensure tracking of the reference signals and meet the LVRT requirements. Figure 3.3 shows the reference calculation and current control loop for the current-controlled DERs.

### 3.4 Proposed Control Structure
The voltage-controlled DERs are becoming more common due to their ability in generating an output voltage that can support the grid and can stabilize a microgrid in islanded conditions. In voltage-controlled DERs, the control system cannot directly control the current. So, the previous approaches discussed in the previous chapter are not applicable. In this section, a control approach is proposed to address this problem.

The dominant operational scenario for the inverter during grid faults is to inject balanced current into the grid while the amount of average real and reactive powers are under control [30, 49]. To develop a control system to realize this situation, we notice that during the grid faults, the voltage at the point of connection is unbalanced, and therefore the controller should be able to generate an unbalanced voltage at the inverter terminals such that the current injected by the inverter during fault remains balanced. To achieve this objective, the voltage controller should be able to regulate both the positive and the negative sequence components of the voltage.

The proposed control structure is shown in Figure 3.4 where the proposed auxiliary controller is shown by the shaded blocks. This controller comprises three major parts: a voltage controller block represented by $UC^+$; the controller $UC^-$ that generates a negative
sequence voltage; and the Sequence Extractor block that extracts the negative sequence component of the current.

To understand the logic behind the proposed method and to derive the internal structure of UC\(^{-}\), we notice that for a balanced positive sequence three-phase system, the abc voltages can be written as

\[ v_a(t) = V \cos(\omega t), \quad v_b(t) = V \cos(\omega t - 120^\circ), \quad v_c(t) = V \cos(\omega t + 120^\circ). \]

In this case, the \(\alpha\beta\) signals may be expressed as

\[ v_\alpha(t) = V \cos(\omega t), \]
\[ v_\beta(t) = V \sin(\omega t). \quad (3.32) \]

The control structure of positive sequence unit, UC\(^{+}\) is shown in Figure 3.5. This structure represents a universal controller which is a voltage controller used for the study in this dissertation. This voltage controller can be replaced by any other voltage based controllers designed to operate for balanced grid situations.
Structure of positive sequence unit ($UC^+$) of proposed controller

To derive the structure for $UC^-$, we notice that in a negatively-sequenced balanced three-phase system, the abc voltages can be written as

\[
\begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} = \begin{bmatrix}
V \cos(\omega t) \\
V \cos(\omega t + 120^\circ) \\
V \cos(\omega t - 120^\circ)
\end{bmatrix}.
\]  

(3.33)

In this case, the $\alpha\beta$ signals are expressed as

\[
\begin{bmatrix}
v_\alpha \\
v_\beta
\end{bmatrix} = \begin{bmatrix}
V \cos(\omega t) \\
-V \sin(\omega t)
\end{bmatrix}.
\]  

(3.34)

Moreover, the variable frequency is identical in both components. Therefore, the controller $UC^-$, has the following differences with $UC^+$. (i) The frequency estimation loop shown by $k_\omega$ is removed and the estimated frequency from $UC^+$ is used. This is for structural simplification. (ii) The parameter $k_q$ is reciprocal of the value used in $UC^+$. (iii) The $\beta$ component uses negative of the sine function. The negative sign of $k_q$ is in order to yield a stable operation and is justified by the stability analysis presented in Chapter 4.
Figure 3.6

Structure of negative sequence unit (UC\(^-\)) of proposed controller

The details of control structure for UC\(^-\) are shown in Figure 3.6. The sequence extractor to extract positive and negative sequences of the current is based on the three-phase EPLL-II structure as explained in Appendix A.

The proposed controller of Figure 3.4 needs four set-points: two for UC\(^+\) that are \(P_{ref}^+\) and \(Q_{ref}^+\) and two for UC\(^-\) that are \(P_{ref}^-\) and \(Q_{ref}^-\). For injection of the balanced positive sequence current, the negative sequence current reference will be zero \((i_{ref}^- = 0)\). So we have \(P_{ref}^- = 0\) and \(Q_{ref}^- = 0\) and consequently, \(P_{ref}^+ = P_{ref}\) and \(Q_{ref}^+ = Q_{ref}\). These reference powers are available for the DERs either locally or from the secondary control through a communication. Therefore, no current or voltage reference generation calculations are required for this control structure. In other words, although the LVRT control is originally defined as a current control problem, the current control is achieved indirectly with the proposed controller and current reference calculations are not required. Calculation of cur-
rent references is computationally heavy as Equation (3.24) shows. This signifies another advantage of the proposed method from computational point of view.

At steady state, the proposed controller supplies a balanced positive sequence current during the unbalanced grid voltage conditions. However, the transient response of the controller is also an important issue. When the fault occurs, there is a sudden change of the grid voltage and this takes a while for the controller to react to it. During this short interval, a large current may flow due to the low impedance of the output filter. This is not acceptable and the seamless transition is desired at the instant of the fault and at the instant when the fault is cleared for the smooth operation of the DERs. Although the proposed controller of Figure 3.4 meets the control objective of balanced current injection, the transients during the transition from the normal operating scenario to the fault conditions and from fault condition to normal operation may not remain in the acceptable range. This is due to the nature of the controller that is a voltage control rather than a current control. The results showing these transients are presented in Chapter 6. These high transients will shut down the inverter due to the violation of the safe operational limits. Consequently, in the next stage of our developments, the proposed control structure is modified to address the transient response issues.

Figure 3.7 shows the structure of the proposed controller where a feed-forward signal is added [40]. To generate the feed-forward signal, a sequence extractor is used which estimates the positive and negative sequence components of the grid voltage. Define $v_f$ as the feed-forward signal, $v_g$, the grid voltage, $\hat{v}_g^+$, the estimated positive sequence component
of the grid voltage, and \( \hat{v}_g^- \), the estimated negative sequence component of the grid voltage.

Then the feed-forward signal is given as

\[
v_f = v_g - \hat{v}_g^+.
\]

(3.35)

If the error term of the sequence extractor is defined as

\[
e = v_g - \hat{v}_g^+ - \hat{v}_g^-.
\]

(3.36)

the feed-forward signal is equivalent to

\[
v_f = v_g - \hat{v}_g^+ = e + \hat{v}_g^-.
\]

(3.37)

The feed-forward signal actually adds this error signal to the controller output. The addition of this error signal improves the transient of the proposed controller. At steady state condition, this error term goes to zero. Therefore, the addition of the feed-forward signal helps to achieve smooth transition from the normal operating conditions to the fault conditions while operating the DERs.

The block diagram of the closed-loop system comprising the proposed controller, the inverter and its PWM switching mechanism, the output interfacing filter and the grid is shown in Figure 3.8. Chapter 4 performs a complete stability analysis of this system for normal and under-fault grid conditions. Performance evaluation of the proposed structure is done in Chapter 6.
Figure 3.7

Proposed control structure with feed-forward signal

Figure 3.8

Block diagram of closed loop system
CHAPTER 4
STABILITY ANALYSIS

This chapter presents a detailed stability analysis of the proposed controller within the grid-connected inverter operation. The stability analysis is first performed using the conventional Jacobian linearization technique. It is observed that the Jacobian method becomes computationally tedious and cannot be conveniently applied to the proposed method in its full version. Therefore, a new linear modeling technique is developed for the stability analysis of the proposed control structure. The model is first verified by comparisons with Jacobian method of individual parts of the proposed controller and then a complete LTI model is developed. The validity of the model is confirmed by various simulations. This LTI model gives valuable information about design of parameters of the controller.

4.1 Stability Analysis using Jacobian Linearization Technique

This section presents stability analysis of the proposed controller using the Jacobian linearization method. The analysis is divided into the positive sequence and negative-sequence units and is done separately. These analyses are used as a basis for verification of the subsequent developed model.
4.1.1 Stability Analysis of Individual Positive Sequence Unit

For the stability analysis of the positive sequence voltage controller unit, a balanced three phase positive sequence voltage is considered. Then, \( abc - \alpha \beta \) transformation is defined as \( v_{\alpha \beta}^+ = v_{\alpha}^+ + jv_{\beta}^+ = \frac{2}{3}(v_{a}^+ + v_{b}^+ e^{j2\pi/3} + v_{c}^+ e^{-j2\pi/3}) \) and \( \alpha \beta - dq \) transformation is defined as \( v_{dq}^+ = v_d^+ + jv_q^+ = e^{-j\theta_g} v_{\alpha \beta}^+ \). The inverter circuit equation is

\[
L \frac{di^+}{dt} = v_{\text{inv}}^+(t) - v_g^+(t) = -Ri^+(t) + v^+(t) - v_g^+(t) \tag{4.1}
\]

where \( v^+(t) \) is the positive sequence virtual inverter voltage generated by the controller. Equation (4.1) is valid in \( abc \) and also in \( \alpha \beta \) domains. The transformation \( x_{dq} = x_d + jx_q = e^{-j\theta_g} x_{\alpha \beta} = e^{-j\theta_g} (x_{\alpha} + jx_{\beta}) \) for any variable \( x \) implies that

\[
\frac{dx_{dq}}{dt} = -j\omega_g x_{dq} + e^{-j\theta_g} \frac{dx_{\alpha \beta}}{dt}. \tag{4.2}
\]

Thus, Equation (4.1) in \( dq \) synchronous reference can be expressed as

\[
\frac{di_d^+}{dt} = -\frac{R}{L} i_d^+ + \omega_d i_q^+ + \frac{1}{L} v_d^+ - \frac{1}{L} v_{gd}^+ \tag{4.3}
\]

\[
\frac{di_q^+}{dt} = -\omega_d i_d^+ - \frac{R}{L} i_q^+ + \frac{1}{L} v_q^+. \tag{4.4}
\]

Recall that the instantaneous real and reactive powers are given by

\[
p^+(t) = \frac{3}{2}[v_d^+ i_d^+ + v_q^+ i_q^+], \tag{4.5}
\]

\[
q^+(t) = \frac{3}{2}[v_q^+ i_d^+ - v_d^+ i_q^+].
\]

From Figure 3.2, the equations defining the voltage controller UC\(^+\) are

\[
\dot{\phi}^+ = \omega + k_p^+ (P'_{\text{ref}} - p^+) \tag{4.6}
\]

\[
\dot{\omega} = k_\omega (P'_{\text{ref}} - p^+). \tag{4.7}
\]
\[ V^+ = k_q (Q'_\text{ref} - q^+) \]  

(4.8)

Now, define the virtual inverter voltage in \(dq\) domain as \(v_d^+ = V^+ \cos \alpha^+\) and \(v_q^+ = V^+ \sin \alpha^+\) where \(V^+ = \sqrt{v_{d_+}^2 + v_{q_+}^2}\) and \(\alpha^+ = \phi^+ - \theta_g = \tan^{-1} \left( \frac{v_{q_+}^+}{v_{d_+}^+} \right)\). Then

\[ \alpha^+ = \omega - \omega_g + k_p P'_\text{ref} - k_p P^+ \]  

(4.9)

\[
v_d^+ = \dot{V}^+ \cos \alpha^+ - \dot{\alpha}^+ V^+ \sin \alpha^+
\]

\[
= [k_q Q'_\text{ref} - \frac{3k_p^+}{2Z} \{ R(v_d^+ i_d^+ + v_q^+ i_q^+) + X(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] \cos \tan^{-1} \frac{v_q^+}{v_d^+}
\]

\[
- [\omega - \omega_g + k_p P'_\text{ref} - \frac{3k_p^+}{2Z} \{ X(v_d^+ i_d^+ + v_q^+ i_q^+) - R(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] v_q^+\]

(4.10)

\[
v_q^+ = \dot{V}^+ \sin \alpha^+ + \dot{\alpha}^+ V \cos \alpha^+
\]

\[
= [k_q Q'_\text{ref} - \frac{3k_p^+}{2Z} \{ R(v_d^+ i_d^+ + v_q^+ i_q^+) + X(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] \sin \tan^{-1} \frac{v_q^+}{v_d^+}
\]

\[
+ [\omega - \omega_g + k_p P'_\text{ref} - \frac{3k_p^+}{2Z} \{ X(v_d^+ i_d^+ + v_q^+ i_q^+) - R(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] v_d^+\]

(4.11)

Equations (4.3), (4.4), (4.7), (4.10), and (4.11) are summarized as

\[
\frac{di_d^+}{dt} = - \frac{R}{L} i_d^+ + \omega_i^q + \frac{1}{L} v_d^+ - \frac{1}{L} v_{gd}^+
\]  

(4.12)

\[
\frac{di_q^+}{dt} = - \omega_g i_d^+ - \frac{R}{L} i_q^+ + \frac{1}{L} v_q^+ - \frac{1}{L} v_{gq}^+
\]  

(4.13)

\[
v_d^+ = [k_q Q'_\text{ref} - \frac{3k_p^+}{2Z} \{ R(v_d^+ i_d^+ + v_q^+ i_q^+) + X(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] \cos \tan^{-1} \frac{v_q^+}{v_d^+} - \frac{3k_p^+}{2Z} \{ X(v_d^+ i_d^+ + v_q^+ i_q^+) - R(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] v_q^+\]

(4.14)

\[
v_q^+ = [k_q Q'_\text{ref} - \frac{3k_p^+}{2Z} \{ R(v_d^+ i_d^+ + v_q^+ i_q^+) + X(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] \sin \tan^{-1} \frac{v_q^+}{v_d^+} + \frac{3k_p^+}{2Z} \{ X(v_d^+ i_d^+ + v_q^+ i_q^+) - R(v_q^+ i_d^+ - v_d^+ i_q^+) \} ] v_d^+\]

(4.15)
\[ \dot{\omega} = k_\omega P'_\text{ref} - \frac{3k_\omega}{2Z} [X(v_d^{+}i_d^{+} + v_q^{+}i_q^{+}) - R(v_q^{+}i_d^{+} - v_d^{+}i_q^{+})] \tag{4.16} \]

Equations (4.12)-(4.16) are nonlinear differential equations. In order to perform a linear stability analysis, equilibrium state of those equations need to be identified. Define the equilibrium point as \((i_d^{*}, i_q^{*}, v_d^{*}, v_q^{*}, \omega^*)\). The linearization of Equations (4.12)-(4.16) about the equilibrium point results in a system expressed as

\[ \dot{X} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{bmatrix} X \tag{4.17} \]

where,

\[ J_{11} = -\frac{R}{L}, \ J_{12} = \omega^*, \ J_{13} = \frac{1}{L}, \ J_{14} = 0, \ J_{15} = 0 \tag{4.18} \]

\[ J_{21} = -\omega^*, \ J_{22} = -\frac{R}{L}, \ J_{23} = 0, \ J_{24} = \frac{1}{L}, \ J_{25} = 0 \tag{4.19} \]

\[ J_{31} = -\frac{3k_q^+}{2Z} \left( Rv_d^{++} + Xv_q^{++} \right) \cos \left( \tan^{-1} \frac{v_q^{++}}{v_d^{++}} \right) \]
\[ + \frac{3k_p^+}{2Z} \left( Xv_d^{++} - Rv_q^{++} \right) v_q^{++} \tag{4.20} \]

\[ J_{32} = -\frac{3k_q^+}{2Z} \left( Rv_q^{++} - Xv_d^{++} \right) \cos \left( \tan^{-1} \frac{v_q^{++}}{v_d^{++}} \right) \]
\[ + \frac{3k_p^+}{2Z} \left( Xv_q^{++} + Rv_d^{++} \right) v_q^{++} \tag{4.21} \]
\[ J_{33} = -\frac{3k^+_q}{2Z} \left( R_i^{+*} - X_i^{+*} \right) \left( \cos \tan^{-1} \frac{v_i^{+*}}{v_d^{+*}} \right) \]
\[ \times \left( k^+_q Q'_{ref} - \frac{3k^+_q}{2Z} \left( R(v_d^{+*}i_d^{+*} + v_q^{+*}i_q^{+*}) + X(v_q^{+*}i_q^{+*} - v_d^{+*}i_d^{+*}) \right) \right) \]
\[ \times \sin \tan^{-1} \frac{v_q^{+*}}{v_d^{+*}} \left( \frac{v_q^{+*}}{V^{+*}} \right) \]
\[ + \frac{3k^+_p}{2Z} \left( X_i^{+*} + R_i^{+*} \right) \left( v_i^{+*} \right) \]
\[ (4.22) \]

\[ J_{34} = -\frac{3k^+_q}{2Z} \left( R_i^{+*} + X_i^{+*} \right) \left( \cos \tan^{-1} \frac{v_i^{+*}}{v_d^{+*}} \right) \]
\[ - \left( k^+_q Q'_{ref} - \frac{3k^+_q}{2Z} \left( R(v_d^{+*}i_d^{+*} + v_q^{+*}i_q^{+*}) + X(v_q^{+*}i_q^{+*} - v_d^{+*}i_d^{+*}) \right) \right) \]
\[ \times \sin \tan^{-1} \frac{v_q^{+*}}{v_d^{+*}} \left( \frac{v_q^{+*}}{V^{+*}} \right) \]
\[ + \frac{3k^+_p}{2Z} \left( X_i^{+*} - R_i^{+*} \right) v_i^{+*} - \left( g^* - \omega_g + k^+_p P'_{ref} \right) \]
\[ + \frac{3k^+_p}{2Z} \left( X(v_d^{+*}i_d^{+*} + v_q^{+*}i_q^{+*}) - R(v_q^{+*}i_q^{+*} - v_d^{+*}i_d^{+*}) \right) \]
\[ (4.23) \]

\[ J_{35} = -v_q^{+*} \]
\[ (4.24) \]

\[ J_{41} = -\frac{3k^+_q}{2Z} \left( Rv_d^{+*} + Xv_q^{+*} \right) \left( \sin \tan^{-1} \frac{v_q^{+*}}{v_d^{+*}} \right) \]
\[ - \frac{3k^+_p}{2Z} \left( Xv_d^{+*} - Rv_q^{+*} \right) \left( v_d^{+*} \right) \]
\[ (4.25) \]

\[ J_{42} = -\frac{3k^+_q}{2Z} \left( Rv_q^{+*} - Xv_d^{+*} \right) \left( \sin \tan^{-1} \frac{v_q^{+*}}{v_d^{+*}} \right) \]
\[ - \frac{3k^+_p}{2Z} \left( Xv_q^{+*} + Rv_d^{+*} \right) \left( v_d^{+*} \right) \]
\[ (4.26) \]
\[
J_{43} = -\frac{3k_p}{2Z} \left( Ri_d^* - X i_q^* \right) \left( \sin^{-1} \frac{v_q^*}{v_d^*} \right) \left( \tan^{-1} \frac{v_q^*}{v_d^*} \right) \\
- \left[ k_q' Q_{\text{ref}}^* - \frac{3k_q}{2Z} \left( R(v_d^{++} i_d^* + v_q^{++} i_q^*) + X(v_q^{++} i_q^* - v_d^{++} i_q^*) \right) \right] \\
\times \cos^{-1} \tan^{-1} \frac{v_q^*}{v_d^*} \left( \frac{v_q^*}{\sqrt{v_d^*}} \right) \\
- \frac{3k_p}{2Z} \left( X i_d^* + R i_q^* \right) v_d^* + \left[ \left( -\omega_g + k_p' P_{\text{ref}}^* \right) \right] \\
- \frac{3k_p}{2Z} \left\{ X(v_d^{++} i_d^* + v_q^{++} i_q^*) - R(v_q^{++} i_q^* - v_d^{++} i_q^*) \right\} 
\tag{4.27}
\]

\[
J_{44} = -\frac{3k_q}{2Z} \left( R i_d^* + X i_q^* \right) \left( \sin^{-1} \frac{v_q^*}{v_d^*} \right) \\
+ \left[ k_q' Q_{\text{ref}}^* - \frac{3k_q}{2Z} \left( R(v_d^{++} i_d^* + v_q^{++} i_q^*) + X(v_q^{++} i_q^* - v_d^{++} i_q^*) \right) \right] \\
\times \cos^{-1} \tan^{-1} \frac{v_q^*}{v_d^*} \left( \frac{v_q^*}{\sqrt{v_d^*}} \right) \\
- \frac{3k_p}{2Z} \left( X i_q^* - R i_d^* \right) v_d^* 
\tag{4.28}
\]

\[
J_{45} = v_d^{++} 
\tag{4.29}
\]

\[
J_{51} = -\frac{3k_p}{2Z} \left( X v_d^{++} - R v_q^{++} \right) 
\tag{4.30}
\]

\[
J_{52} = -\frac{3k_p}{2Z} \left( X v_q^{++} + R v_d^{++} \right) 
\tag{4.31}
\]

\[
J_{53} = -\frac{3k_p}{2Z} \left( X i_d^* + R i_q^* \right) 
\tag{4.32}
\]

\[
J_{54} = -\frac{3k_p}{2Z} \left( X i_q^* - R i_d^* \right) 
\tag{4.33}
\]

\[
J_{55} = 0. 
\tag{4.34}
\]

For the operation at \( P_{\text{ref}} = 0, \; Q_{\text{ref}} = 0 \), the equilibrium state is identified as

\[
(i_d^{++}, i_q^{++}, v_d^{++}, v_q^{++}, w^*) = (0, 0, V^+, 0, w_g). 
\]
Then, the following system is obtained from Equation (4.17).

\[
\dot{X} = \begin{bmatrix}
  -\frac{R}{L} & \omega_g & \frac{1}{L} & 0 & 0 \\
  -\omega_g & -\frac{R}{L} & 0 & \frac{1}{L} & 0 \\
  -\frac{3Rk_+^+V^+}{2Z} & -\frac{3Xk_+^+V^+}{2Z} & 0 & 0 & 0 \\
  -\frac{3Xk_+^+V^+}{2Z} & -\frac{3Rk_+^+V^+}{2Z} & 0 & 0 & V^+ \\
  \frac{3Xk_+^+V^+}{2Z} & -\frac{3Rk_+^+V^+}{2Z} & 0 & 0 & 0 \\
\end{bmatrix} X 
\]  \hspace{1cm} (4.35)

This matrix depends on the inverter and grid parameters \((L, V^+, \omega)\) and the controller parameters \((R, k_p^+, k_q^+, k_\omega)\). Eigenvalues of this matrix determine the stability of the system. Based on the elements of the jacobian matrix of (4.35), the controller parameters are selected as, \(k_q^+ = \frac{k_q^+}{V^+}\) and \(k_p^+ = \frac{k_p^+}{V^+}\). The gain for frequency estimation branch is selected as \(k_\omega = \frac{k_\omega^+}{\sqrt{2}V^+}\) for a constant \(k^+ > 0\) and a damping ratio \(\zeta\). Then, the design is reduced to the design of two gains \(R\) and \(k^+\). The damping ratio \(\zeta\) is selected between \(\frac{\sqrt{2}}{2}\) to \(\sqrt{2}\).

To design the parameters \(R\) and \(k^+\), we notice that in a grid-connected operation, we can disable the frequency loop gain of the controller by setting \(k_\omega = 0\). This reduces the Equation (4.35) to the \(4 \times 4\) matrix given as

\[
\dot{X} = \begin{bmatrix}
  -\frac{R}{L} & \omega_g & \frac{1}{L} & 0 \\
  -\omega_g & -\frac{R}{L} & 0 & \frac{1}{L} \\
  -\frac{3Rk_+^+V^+}{2Z} & -\frac{3Xk_+^+V^+}{2Z} & 0 & 0 \\
  \frac{3Xk_+^+V^+}{2Z} & -\frac{3Rk_+^+V^+}{2Z} & 0 & 0 \\
\end{bmatrix} X. 
\] \hspace{1cm} (4.36)

The characteristic equation of the system of Equation (4.36) is

\[
s^4 + \frac{2R}{L} s^3 + \frac{Z^3 + 3RLk_+^+}{L^2Z} s^2 + \frac{3Zk_+^+}{L^2} s + \frac{2.25k_+^+}{L^2} = 0. \hspace{1cm} (4.37)
\]
This system has four poles. Our investigations show that if $k^+$ is selected such that all four poles have identical real-parts, then the system exhibits a desirable response. Consider $-\alpha$ as the identical real part of all four eigenvalues of the system of Equation (4.36). Then the roots of the system are $s = -\alpha \pm j\beta$ and $s = -\alpha \pm j\gamma$ where $\beta$ and $\gamma$ are the imaginary parts of the eigenvalues. Then the characteristic equation of the system will be

$$s^4 + 4\alpha s^3 + (6\alpha^2 + \beta^2 + \gamma^2)s^2 + \{4\alpha^3 + 2\alpha(\beta^2 + \gamma^2)\}s + \{\alpha^4 + \alpha^2(\beta^2 + \gamma^2) + \beta^2\gamma^2\} = 0.$$  \hspace{1cm} (4.38)

Comparing Equations (4.37) and (4.38), which are the identical equations, the values for the parameters $R$ and $k^+$ are formulated as

$$R = 2\alpha L, \quad k^+ = \frac{1}{3} \frac{RZ}{L}.$$  \hspace{1cm} (4.39)

Therefore, these values of the parameters $R$ and $k^+$ will place the four eigenvalues of the system of Equation (4.36) at a position with identical real part of $-\alpha$. Substituting Equation (4.39) in Equation (4.37) results in

$$s^4 + 4\alpha s^3 + (8\alpha^2 + \omega^2)s^2 + 2\alpha(4\alpha^2 + \omega^2)s + \alpha^2(4\alpha^2 + \omega^2) = 0.$$  \hspace{1cm} (4.40)

The analysis of characteristic equation expressed in Equation (4.40) concludes that, for a given grid frequency ($\omega$), the system is stable for $\alpha > 0$.

The eigenvalues of the system of Equation (4.35) when $R = 2\alpha L$ and $k^+$ is increased from 0 to $k^+ = \frac{1}{3} \frac{RZ}{L}$ are shown in Figure 4.1 for three different values of $L$ (2, 5 and 10 mH) and three different values of $\alpha$ (50, 100 and 150). The grid is 60 Hz, 120 V (rms, line-to-neutral). The system has two eigenvalues at $-\frac{R}{L} \pm j\omega$ and three eigenvalues at 0.
Figure 4.1

Eigenvalues of the UC$^+$ control unit ($k^+$ changes from 0 to $\frac{RZ}{3L}$)

when $k^+ = 0$. Among them, there are four which show fast modes and one slow that corresponds to the frequency. This study shows that the fast modes will have an identical real part approximately at $-\alpha$ and this confirms the above derivations. Accordingly, we
Eigenvalues of the UC$^+$ control unit when the parameter $k^+$ varies can summarize the root-locus as shown in Figure 4.2 for arbitrary values of $L$ and $\alpha$ within our practical range of interest.

The above stability analysis is performed for the operating point at $P_{\text{ref}} = 0$, $Q_{\text{ref}} = 0$. When the inverter operates at a nonzero power, similar analysis can be done. Our studies show that the same design setting of Equation (4.39) for zero power can respond adequately to non-zero operations. Figure 4.3 shows the impact of nonzero power operations on the eigenvalues of the system when the above proposed parameter setting is used. It confirms that the eigenvalues remain reasonably robust within an entire range of operation.
Eigenvalues of the UC$^+$ control unit for non-zero powers ($L = 5$ mH, $\alpha = 100$)

4.1.2 Stability Analysis of Individual Negative Sequence Unit

The voltage and current parameters used in this section are the negative sequence components. The $abc - \alpha\beta$ transformation is defined as $v_{\alpha\beta} = v_\alpha^- + jv_\beta^- = \frac{2}{3}(v_a^- + v_b^- e^{j2\pi/3} + v_c^- e^{-j2\pi/3})$ and $\alpha\beta - dq$ transformation is defined as $v_{dq}^- = v_d^- - jv_q^- = e^{j\theta}v_{\alpha\beta}^-$ for the negative sequence components. Equations in $dq$ synchronous reference are then expressed as

$$\frac{di_d^-}{dt} = -\frac{R}{L}i_d^- - \omega_qi_q^- + \frac{1}{L}v_d^- - \frac{1}{L}v_{gd}^-$ (4.41)
The negative sequence components of the virtual inverter voltages are, \( v_d^- = V^- \cos \alpha^- \) and \( v_q^- = -V^- \sin \alpha^- \) where \( V^- = \sqrt{v_d^2 + v_q^2} \) and \( \alpha^- = (\phi^- - \theta_g) = \tan^{-1}\left(\frac{v_q^-}{v_d^-}\right) \).

Then
\[
\mathbf{X}^\prime \mathbf{X} = \mathbf{X} \mathbf{X} \]

The dynamics of the frequency loop is expressed as
\[
\dot{\omega} = k_w P'_\text{ref} - \frac{3k_w}{2Z} [X(v_d^- i_d^- + v_q^- i_q^-) - R(v_q^- i_d^- - v_d^- i_q^-)].
\] (4.45)

The linearization of Equations (4.41)-(4.45) about the equilibrium point defined as \((i_d^{*-\star}, i_q^{*-\star}, v_d^{*-\star}, v_q^{*-\star}, \omega^*)\) results in a system expressed as
\[
\mathbf{X}^- = \begin{bmatrix}
J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\
J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\
J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\
J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\
J_{51} & J_{52} & J_{53} & J_{54} & J_{55}
\end{bmatrix} \mathbf{X}^- \] (4.46)
where,

\[
J_{11} = -\frac{R}{L}, \quad J_{12} = -\omega^*, \quad J_{13} = \frac{1}{L}, \quad J_{14} = 0, \quad J_{15} = 0
\]  \quad (4.47)

\[
J_{21} = \omega^*, \quad J_{22} = -\frac{R}{L}, \quad J_{23} = 0, \quad J_{24} = \frac{1}{L}, \quad J_{25} = 0
\]  \quad (4.48)

\[
J_{31} = -\frac{3k_q^-}{2Z} \left( \dot{r}_{d^*} + Xv_{q^*} \right) \cos \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-}
\]
\[
- \frac{3k_p^-}{2Z} \left( \dot{r}_{d^*} - Rv_{q^*} \right) V_{d^*}^\prime
\]  \quad (4.49)

\[
J_{32} = -\frac{3k_q^-}{2Z} \left( \dot{r}_{q^*} - Xv_{d^*} \right) \cos \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-}
\]
\[
- \frac{3k_p^-}{2Z} \left( \dot{r}_{q^*} + Rv_{d^*} \right) V_{d^*}^\prime
\]  \quad (4.50)

\[
J_{33} = -\frac{3k_q^-}{2Z} \left( \dot{r}_{d^*} - Xv_{q^*} \right) \cos \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-}
\]
\[
- \left[ k^q Q'_{\text{ref}} - \frac{3k_q^-}{2Z} \left( R(v_{d^*}^- i_{d^*}^- + v_{q^*}^- i_{q^*}^-) + X(v_{q^*}^- i_{d^*}^- - v_{d^*}^- i_{q^*}^-) \right) \right]
\]
\[
\times \sin \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-} \right) \frac{v_{q^*}^-}{V_{d^*}^\prime}
\]

\[
- \frac{3k_p^-}{2Z} \left( \dot{r}_{q^*} + Rv_{d^*} \right) i_{q^*}^\prime
\]  \quad (4.51)

\[
J_{34} = -\frac{3k_q^-}{2Z} \left( \dot{r}_{q^*} + Xv_{d^*} \right) \cos \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-}
\]
\[
+ \left[ k^q Q'_{\text{ref}} - \frac{3k_q^-}{2Z} \left( R(v_{d^*}^- i_{d^*}^- + v_{q^*}^- i_{q^*}^-) + X(v_{q^*}^- i_{d^*}^- - v_{d^*}^- i_{q^*}^-) \right) \right]
\]
\[
\times \sin \tan^{-1} \frac{-v_{q^*}^-}{v_{d^*}^-} \right) \frac{v_{d^*}^-}{V_{d^*}^\prime}
\]

\[
- \frac{3k_p^-}{2Z} \left( \dot{r}_{q^*} - Rv_{d^*} \right) i_{q^*}^\prime + \left( \dot{q}^* - \omega_g + k_p^- P'_{\text{ref}} \right)
\]
\[
- \frac{3k_p^-}{2Z} \left( X(v_{d^*}^- i_{d^*}^- + v_{q^*}^- i_{q^*}^-) - R(v_{q^*}^- i_{d^*}^- - v_{d^*}^- i_{q^*}^-) \right)
\]  \quad (4.52)

\[
J_{35} = v_{q^*}^\prime
\]  \quad (4.53)
\[ J_{41} = \frac{3k^p_q}{2Z} (Rv^{*-}_d + Xv^{*-}_q) \left( \sin^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \frac{3k^p_p}{2Z} (Xv^{*-}_d - Rv^{*-}_q) v^{*-}_d \]  
(4.54)

\[ J_{42} = \frac{3k^p_q}{2Z} (Rv^{*-}_q - Xv^{*-}_d) \left( \sin^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \frac{3k^p_p}{2Z} (Xv^{*-}_q + Rv^{*-}_d) v^{*-}_d \]  
(4.55)

\[ J_{43} = + \frac{3k^p_q}{2Z} (Ri^{*-}_d - Xi^{*-}_q) \left( \sin^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \left[ k^p_q Q'_{ref} - \frac{3k^p_q}{2Z} \left\{ R(v^{*-}_d i^{*-}_d + v^{*-}_q i^{*-}_q) + X(v^{*-}_q i^{*-}_d - v^{*-}_d i^{*-}_q) \right\} \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ \times \cos \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \frac{3k^p_p}{2Z} (X i^{*-}_d + R i^{*-}_q) v^{*-}_d \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \frac{3k^p_p}{2Z} \left\{ X(v^{*-}_d i^{*-}_d + v^{*-}_q i^{*-}_q) - R(v^{*-}_q i^{*-}_d - v^{*-}_d i^{*-}_q) \right\} \]  
(4.56)

\[ J_{44} = + \frac{3k^p_q}{2Z} (R i^{*-}_q + X i^{*-}_d) \left( \sin^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \left[ k^p_q Q'_{ref} - \frac{3k^p_q}{2Z} \left\{ R(v^{*-}_d i^{*-}_d + v^{*-}_q i^{*-}_q) + X(v^{*-}_q i^{*-}_d - v^{*-}_d i^{*-}_q) \right\} \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ \times \cos \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
\[ + \frac{3k^p_p}{2Z} (X i^{*-}_q - R i^{*-}_q) v^{*-}_d \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \left( \tan^{-1} \frac{-v^{*-}_q}{v^{*-}_d} \right) \]  
(4.57)

\[ J_{45} = -v^{*-}_d \]  
(4.58)

\[ J_{51} = -\frac{3k^p}{2Z} (Xv^{*-}_d - Rv^{*-}_q) \]  
(4.59)

\[ J_{52} = -\frac{3k^p}{2Z} (Xv^{*-}_d + Rv^{*-}_q) \]  
(4.60)

\[ J_{53} = -\frac{3k^p}{2Z} (X i^{*-}_d + R i^{*-}_q) \]  
(4.61)
\[ J_{54} = -\frac{3k_\omega}{2Z} (X_i^* - R i_d^*) \]  
\[ J_{55} = 0. \]  
\[ (4.62) \]
\[ (4.63) \]

For grid-connected operation (consider negative sequence grid) at \( P_{\text{ref}} = 0, \ Q_{\text{ref}} = 0, \)
the equilibrium point is

\[ (i_d^*, i_q^*, v_d^*, v_q^*, w^*) = (0, 0, V^-, 0, w_g). \]

Then, the following linear system is obtained from Equation 4.46.

\[ \dot{X}^- = \begin{bmatrix} -\frac{R}{L} & -\omega_g & \frac{1}{L} & 0 & 0 \\ \omega_g & -\frac{R}{L} & 0 & \frac{1}{L} & 0 \\ -\frac{3Rk_pV^-}{2Z} & \frac{3Xk_pV^-}{2Z} & 0 & 0 & 0 \\ \frac{3Xk_pV^-}{2Z} & \frac{3Rk_pV^-}{2Z} & 0 & 0 & -V^- \end{bmatrix} X^- \]  
\[ (4.64) \]

The control parameters for the negative-sequence unit, are selected as \( k_p^- = \frac{k^-}{V^-}, k_q^- = -\frac{k^-}{V^-}. \) Moreover, \( R \) and \( k^- \) are related through following equations.

\[ R = 2\alpha L, \ k^- = \frac{1}{3} \frac{RZ}{L}. \]  
\[ (4.65) \]

The frequency loop for negative sequence control unit can be disabled (\( k_\omega = 0 \)) as the
frequency estimated by positive sequence control unit is used in this control loop. Then,
the Equation (4.64) reduces to following system.

\[ \dot{X}^+ = \begin{bmatrix} -\frac{R}{L} & \omega_g & \frac{1}{L} & 0 \\ -\omega_g & -\frac{R}{L} & 0 & \frac{1}{L} \\ \frac{3Rk^-}{2Z} & \frac{3Xk^-}{2Z} & 0 & 0 \\ \frac{3Xk^-}{2Z} & \frac{3Rk^-}{2Z} & 0 & 0 \end{bmatrix} X. \]  
\[ (4.66) \]
The characteristic equation of the system of Equation (4.66) is

\[ s^4 + \frac{2R}{L} s^3 + \frac{Z^3 - 3RLk^-}{L^2Z} s^2 + \frac{3k^- (X^2 - R^2)}{ZL^2} s + \frac{2.25k^-^2}{L^2} = 0. \]  

(4.67)

Substituting Equation (4.65) in Equation (4.67) results in

\[ s^4 + 4\alpha s^3 + \omega^2 s^2 + 2\alpha (-4\alpha^2 + \omega^2) s + \alpha^2 (4\alpha^2 + \omega^2) = 0. \]  

(4.68)

The analysis of characteristic equation expressed in Equation (4.68) concludes that, for a given grid frequency (\(\omega\)), the system is stable for \(0 < \alpha < (\omega/\sqrt{8})\).

Figure 4.4 shows the closed-loop poles of negative sequence unit. This figure shows that the loop is stable and the fast poles will have an almost identical real part at \(-\alpha = -50\).

Figure 4.5 shows the root-locus of negative-sequence controller for different values of \(RZ\) and \(\alpha\) when \(k^-\) varies from 0 to \(\frac{1}{3} \frac{RZ}{L}\). It is observed that the negative sequence controller
Figure 4.5

Eigenvalues of the UC\(^{-}\) control unit (\(k^{-}\) changes from 0 to \(\frac{RZ}{3L}\))

unit is sensitive to high values of \(\alpha\). However, it appears that it still works for \(\alpha\) as large as 100 which can practically address the control requirements in terms of response transient times.
4.2 Stability Analysis of Proposed Controller using LTI Model

This section provides the stability analysis of the proposed controller using an alternative linear modeling method which was first presented in [24] for stability analysis of universal controller for single phase inverter. Concept of this linear modeling technique is extended for the stability analysis of positive sequence unit, negative sequence unit, and the combined positive/negative components of the proposed controller in this section. The LTI model developed here gives valuable information about design of parameters of the controller.

4.2.1 Stability Analysis of Individual Positive Sequence Unit

Define the virtual inverter voltage in \( \alpha\beta \) frame as the state vector \( x(t) \) given by

\[
x(t) = \begin{pmatrix} v^+_\alpha \\ v^+_\beta \end{pmatrix} = \begin{pmatrix} V^+ \\ \cos \phi^+ \\ \sin \phi^+ \end{pmatrix}
\]  

(4.69)

\[
\dot{x} = \begin{pmatrix} \dot{v}^+_\alpha \\ \dot{v}^+_\beta \end{pmatrix} = \begin{pmatrix} \dot{V}^+ \cos \phi^+ - \dot{\phi}^+ V^+ \sin \phi^+ \\ \dot{V}^+ \sin \phi^+ + \dot{\phi}^+ V^+ \cos \phi^+ \end{pmatrix}
\]  

(4.70)

The positive sequence voltage controller (UC\(^+\)) equations are expressed as

\[
\frac{d}{dt} \begin{pmatrix} \delta(t) \\ V(t) \end{pmatrix} = \begin{pmatrix} k_p & 0 \\ 0 & k_i \end{pmatrix} Y(\theta) \begin{pmatrix} P_{\text{ref}} - p(t) \\ Q_{\text{ref}} - q(t) \end{pmatrix}
\]  

(4.71)
where

\[ T(\theta) = \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} = \frac{1}{Z} \begin{pmatrix} X & -R \\ R & X \end{pmatrix} \]  
(4.72)

Now, we use the expression

\[ \begin{pmatrix} V^+ \dot{\phi}^+ \\ V^+ \dot{\omega} \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{V}^+ \\ V^+ \end{pmatrix} \]  
(4.73)

and

\[ \begin{pmatrix} V^+ \delta^+ \\ \dot{V}^+ \end{pmatrix} = \begin{pmatrix} (V^+ \delta^+) \left( \begin{pmatrix} V^+ k_p^+ & 0 \\ 0 & k_q^+ \end{pmatrix} \right) Y(\theta) \begin{pmatrix} P_{\text{ref}} - p^+ \\ Q_{\text{ref}} - q^+ \end{pmatrix} \end{pmatrix} \]  
(4.74)

where

\[ \begin{pmatrix} p^+ \\ q^+ \end{pmatrix} = \frac{3}{2} \begin{pmatrix} v^\alpha^+ & v^\beta^+ \\ v^\beta^- & v^\alpha^- \end{pmatrix} \begin{pmatrix} i^\alpha^+ \\ i^\beta^+ \end{pmatrix} = \frac{3}{2} V^+ \begin{pmatrix} \cos \phi^+ & \sin \phi^+ \\ \sin \phi^+ & -\cos \phi^+ \end{pmatrix} \begin{pmatrix} i^\alpha^+ \\ i^\beta^+ \end{pmatrix} \]

Then, the UC control unit reduces to the LTI system if we select the gains \( k_p^+ = \frac{k^+}{V^+} \), \( k_q^+ = \frac{k^+}{V^+} \) for a constant \( k^+ \). The proof is by substituting these expressions to achieve

\[ \begin{pmatrix} V^+ \delta^+ \\ \dot{V}^+ \end{pmatrix} = \begin{pmatrix} (V^+ k_p^+) & 0 \\ 0 & k_q^+ \end{pmatrix} \begin{pmatrix} P'_{\text{ref}} \\ Q'_{\text{ref}} \end{pmatrix} \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} \begin{pmatrix} \sin(\theta - \phi^+) & \cos(\theta - \phi^+) \\ \cos(\theta - \phi^+) & \sin(\theta - \phi^+) \end{pmatrix} \begin{pmatrix} i^\alpha^+ \\ i^\beta^+ \end{pmatrix} \]  
(4.75)
Now,

\[ \dot{x} = \begin{pmatrix} -\sin \phi^+ & \cos \phi^+ \\ \cos \phi^+ & \sin \phi^+ \end{pmatrix} \begin{pmatrix} V^+ \phi^+ \\ V^+ \end{pmatrix} = \begin{pmatrix} -\sin \phi^+ & \cos \phi^+ \\ \cos \phi^+ & \sin \phi^+ \end{pmatrix} \begin{pmatrix} V^+ \phi^+ \\ 0 \end{pmatrix} \begin{pmatrix} -\sin \phi^+ & \cos \phi^+ \\ \cos \phi^+ & \sin \phi^+ \end{pmatrix} \begin{pmatrix} k^+ \frac{P'_{\text{ref}}}{V^+} \\ \frac{Q'_{\text{ref}}}{V^+} \end{pmatrix} \]

\[ = -\frac{3}{2} k^+ \begin{pmatrix} -\sin \phi^+ & \cos \phi^+ \\ \cos \phi^+ & \sin \phi^+ \end{pmatrix} \begin{pmatrix} \sin(\theta - \phi^+) & \cos(\theta - \phi^+) \\ \cos(\theta - \phi^+) & -\sin(\theta - \phi^+) \end{pmatrix} \begin{pmatrix} i^+_{\alpha} \\ i^+_{\beta} \end{pmatrix} \]

which gives

\[ \dot{x} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} v^+_{\alpha} \\ v^+_{\beta} \end{pmatrix} + \frac{k^+}{V^{+2}} \begin{pmatrix} Q'_{\text{ref}} - P'_{\text{ref}} \\ P'_{\text{ref}} Q'_{\text{ref}} \end{pmatrix} \begin{pmatrix} i^+_{\alpha} \\ i^+_{\beta} \end{pmatrix} - \frac{3}{2} \frac{k^+}{V^{+2}} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} i^+_{\alpha} \\ i^+_{\beta} \end{pmatrix} \]

Finally, the state equation can be expressed as

\[ \dot{x} = \Omega + \frac{k^+}{V^{+2}} \Gamma \begin{pmatrix} v \\ -\frac{3}{2} k^+ \Theta y \end{pmatrix} \]

where

\[ \Omega = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} Q'_{\text{ref}} - P'_{\text{ref}} \\ P'_{\text{ref}} Q'_{\text{ref}} \end{pmatrix}, \quad \Theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \]

Equation (4.78) shows the state equation. This model is nearly linear if the positive sequence virtual inverter voltage magnitude \( V^+ \) is assumed constant or nearly constant.
For fault conditions, this model can be used for stability analysis prior to the fault and after
the fault by modifying the matrix $\Gamma$ and the variable $V^+$ appropriately.

The control block diagram of closed loop system based on this model used for the
stability analysis is shown in Figure 4.6. The closed loop system is expressed as

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \Omega + \frac{k^+}{V+\omega^2} \Gamma & -\frac{3}{2}k^+\Theta \\ \frac{1}{L}I_2 & -\frac{R}{L}I_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} O_2 \\ \frac{1}{L}I_2 \end{pmatrix} v_g $$

where $O_2$ and $I_2$ are $2 \times 2$ zero and identity matrices respectively.

For an exemplary case of $P_{\text{ref}} = 0$ and $Q_{\text{ref}} = 0$, the eigenvalues of the system are
plotted by changing $k^+$ as shown in Figure 4.7 for three different values of $L$ (2, 5 and
10 mH) and $\alpha$ (50, 100 and 150). For $k^+ = 0$ the two eigenvalues of the system are at $-\frac{R}{L}$
and the two at $\pm j\omega$. The four eigenvalues have identical real parts at $-\alpha$ when $R = 2\alpha L$
and $k^+ = \frac{1}{3} \frac{RZ}{L}$. This is in perfect agreement with the results of Jacobian linearization
method discussed in Section 4.1.1. Figure 4.8 shows the eigenvalues of the system for
arbitrary values of $L$ and $\alpha$ within our practical range of interest when $k^+$ changes from 0
to $\frac{1}{3} \frac{RZ}{L}$. 

55
The above stability analysis is performed for the operating point at $P_{\text{ref}} = 0$, $Q_{\text{ref}} = 0$. When the inverter operates at a nonzero power, similar analysis can be done. Figure 4.9 shows the impact of nonzero power operations on the eigenvalues of the system. It confirms that the eigenvalues remain reasonably robust within an entire range of operation.
Eigenvalues of the LTI model of UC+ unit when the parameter $k^+$ varies

The eigenvalues of the Jacobian matrix (shown in Figure 4.2) are $j\omega$ shifted versions of actual eigenvalues of the system due to dq-transformation. Figure 4.8 shows the actual eigenvalues without such a shift. In other words, the LTI model developed here and shown in Figure 4.6 represents the system in actual time domain frame.

Another remark should be made regarding the number of eigenvalues. The Jacobian matrix has five eigenvalues while the system of Figure 4.6 has four eigenvalues. The reason is that the dynamics of the variable frequency, $\omega$, is not included in the model of Figure 4.6. However, it is possible to characterize that dynamics as follows. Based on the EPLL theory [22, 23], the eigenvalue of the frequency loop is $(-\zeta + \sqrt{\zeta^2 - 1}) \frac{k}{k_{\omega}}$ when the frequency loop gain is $k_\omega = \frac{k^+ \zeta^2}{\kappa V^2}$. Here, $\zeta$ is the damping ratio whose value is given
Figure 4.9

Eigenvalues of the LTI model of UC+ control unit for non-zero powers ($L = 5$ mH, $\alpha = 100$)

as $\frac{\sqrt{2}}{2} \leq \zeta \leq \sqrt{2}$. For our study, $\zeta = \sqrt{2}$ is selected which gives $k_{\omega} = \frac{k^{+2}}{4V^2\tau}$. Figure 4.10 shows the frequency eigenvalue of the Jacobian matrix and the value of $(-\zeta + \sqrt{\zeta^2 - 1})\frac{k^{+}}{4\zeta}$ when $k^{+}$ varies. This figure confirms accuracy of this location for the frequency pole.
4.2.2 Stability Analysis of Individual Negative Sequence Unit

Similar to the positive sequence analysis, an analysis and LTI model can be developed for the negative-sequence unit. The negative sequence component of the inverter voltage in $\alpha\beta$ frame defined as a state vector $x(t)$ is given by

$$x(t) = \begin{pmatrix} v^-_\alpha \\ v^-_\beta \end{pmatrix} = \begin{pmatrix} \cos \phi^- \\ -\sin \phi^- \end{pmatrix}$$

(4.80)

Then,

$$\dot{x} = \begin{pmatrix} V^- \cos \phi^- - \dot{\phi}^- V^- \sin \phi^- \\ -V^- \sin \phi^- - \dot{\phi}^- V^- \cos \phi^- \end{pmatrix} = \begin{pmatrix} -\sin \phi^- & \cos \phi^- \\ -\cos \phi^- & -\sin \phi^- \end{pmatrix} \begin{pmatrix} V^- \phi^- \\ V^- \end{pmatrix}$$

(4.81)

and

$$\begin{pmatrix} (V^- \phi^-) \\ \dot{V}^- \end{pmatrix} = \begin{pmatrix} V^- \omega \\ 0 \end{pmatrix} + \begin{pmatrix} V^- \delta^- \\ \dot{V}^- \end{pmatrix}$$

(4.82)
The UC control unit reduces to an LTI system if we select the gains $k_p = \frac{k^-}{V^-}$, $k_q = -\frac{k^-}{V^-}$ for a constant $k^-$. The proof is summarized in the following few steps. First, we notice that

$$
\begin{pmatrix}
V^- \delta^- \\
V^-
\end{pmatrix}
= 
\begin{pmatrix}
V^-k_p & 0 \\
0 & k_q
\end{pmatrix}
\begin{pmatrix}
(\mathbf{I}(\theta)) \\
(-p^-)
\end{pmatrix}
$$

$$
= -3k^- \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\sin \theta & -\cos \theta \\
\cos \theta & \sin \theta
\end{pmatrix}
\begin{pmatrix}
\cos \phi^- & -\sin \phi^- \\
-\sin \phi^- & \cos \phi^-
\end{pmatrix}
\begin{pmatrix}
i_{\alpha}^- \\
i_{\beta}^-
\end{pmatrix}
$$

$$
= -3 \frac{k^-}{2}
\begin{pmatrix}
\sin(\theta + \phi^-) & \cos(\theta + \phi^-) \\
-\cos(\theta + \phi^-) & \sin(\theta + \phi^-)
\end{pmatrix}
\begin{pmatrix}
i_{\alpha}^- \\
i_{\beta}^-
\end{pmatrix}
$$

(4.83)

Now,

$$
\dot{x} = 
\begin{pmatrix}
-\sin \phi^- & \cos \phi^- \\
-\cos \phi^- & -\sin \phi^-
\end{pmatrix}
\begin{pmatrix}
V^- \dot{\phi}^- \\
V^-
\end{pmatrix}
$$

$$
= 
\begin{pmatrix}
-\sin \phi^- & \cos \phi^- \\
-\cos \phi^- & -\sin \phi^-
\end{pmatrix}
\begin{pmatrix}
V^- \omega \\
0
\end{pmatrix}
$$

$$
-3 \frac{k^-}{2}
\begin{pmatrix}
-\sin \phi^- & \cos \phi^- \\
-\cos \phi^- & -\sin \phi^-
\end{pmatrix}
\begin{pmatrix}
\sin(\theta + \phi^-) & \cos(\theta + \phi^-) \\
-\cos(\theta + \phi^-) & \sin(\theta + \phi^-)
\end{pmatrix}
\begin{pmatrix}
i_{\alpha}^- \\
i_{\beta}^-
\end{pmatrix}
$$

$$
= \begin{pmatrix}
0 & \omega \\
-\omega & 0
\end{pmatrix}
\begin{pmatrix}
v_{\alpha}^- \\
v_{\beta}^-
\end{pmatrix}
+ 3 \frac{k^-}{2}
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
i_{\alpha}^- \\
i_{\beta}^-
\end{pmatrix}
$$

(4.84)

Finally, the state equation can be expressed as

$$
\dot{x} = -\Omega x + \frac{3}{2} k^- \Theta y
$$

(4.85)
Eigenvalues of UC\(^-\) unit when the parameter \(k^-\) varies (for \(-\alpha = -50\))

where

\[
\Omega = \begin{pmatrix}
0 & -\omega \\
\omega & 0
\end{pmatrix}, \quad \Theta = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}, \\
y = \begin{pmatrix}
i_{\alpha}^- \\
i_{\beta}^-
\end{pmatrix}
\]

The state-space representation of the closed loop system for the stability analysis is expressed as

\[
\frac{d}{dt} \begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
-\Omega & \frac{3}{2} k^- \Theta \\
\frac{1}{L} I_2 & -\frac{R}{L} I_2
\end{pmatrix} \begin{pmatrix}
x \\
y
\end{pmatrix} + \begin{pmatrix}
O_2 \\
\frac{1}{L} I_2
\end{pmatrix} \delta_g \tag{4.86}
\]

where \(O_2\) and \(I_2\) are \(2 \times 2\) zero and identity matrices respectively.
Figure 4.12

Eigenvalues of the LTI model of UC$^{-}$ control unit ($k^-$ changes from 0 to $\frac{RZ}{3L}$)

For $P_{\text{ref}} = 0$ and $Q_{\text{ref}} = 0$, the eigenvalues of the system plotted by changing $k^-$ is shown in Figure 4.11 for $\alpha = 50$ where the four eigenvalues have identical real parts at $-\alpha$ when $R = 2\alpha L$ and $k^- = \frac{RZ}{3L}$.

Figure 4.12 shows the eigenvalues of the system for different values of $L$ and $\alpha$ when $k^-$ varies form 0 to $\frac{RZ}{3L}$. This is in perfect agreement with the results of Jacobian lineariza-
tion method discussed in Section 4.1.2. We notice here again that there is a $j\omega$ shift in the eigenvalues of Jacobian matrix as compared to the actual eigenvalues of the system.

### 4.2.3 Complete Modeling of Proposed Controller

In Section 4.2.1, the LTI model of UC$^+$ control unit was developed for the stability analysis. This model was also validated using the Jacobian linearization technique in Section 4.1.1. Similarly, an LTI model for UC$^-$ control unit was developed and validated by Jacobian linearization technique. In addition to the UC$^+$ and UC$^-$ control units, the proposed controller also consists of the sequence extractor unit. The LTI model of the sequence extractor unit is developed in [22, Chapter 8] and is reviewed in Appendix A. The closed loop control model of the entire system for the complete stability analysis of the proposed controller is thus developed and shown in Figure 4.13. This diagram is two-phase in $\alpha\beta$ domain. Further details are explained as following.

![Proposed LTI model of entire closed loop system for stability analysis](image)

\[ y_4 = \frac{1}{Ls+R} y_4 \]

Figure 4.13

Proposed LTI model of entire closed loop system for stability analysis
Define the state vector \( x_1 = \begin{pmatrix} v_{\alpha}^- \\ v_{\beta}^- \end{pmatrix} \), input \( u_1 = \begin{pmatrix} i_{\alpha} \end{pmatrix} \), and output \( y_1 = \begin{pmatrix} i_{\beta} \end{pmatrix} \). Then, the linear model of UC\(^+\) unit is

\[
\dot{x}_1 = A_1 x_1 + B_1 y_4
\]
\[y_1 = C_1 x_1 \tag{4.87}\]

where

\[A_1 = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} + \frac{k^+}{V^2} \begin{pmatrix} Q_{\text{ref}}' & -P_{\text{ref}}' \\ P_{\text{ref}}' & Q_{\text{ref}}' \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[B_1 = -\frac{3}{2} k^+ \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

Notice that for \( P_{\text{ref}} = 0 \) and \( Q_{\text{ref}} = 0 \), \( A_1 = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \).

Similarly, for the UC\(^-\) unit, define state \( x_2 = \begin{pmatrix} v_{\alpha}^- \\ v_{\beta}^- \end{pmatrix} \), input \( u_2 = \begin{pmatrix} i_{\alpha} \\ i_{\beta} \end{pmatrix} \), and output \( y_2 = \begin{pmatrix} v_{\alpha}^- \\ v_{\beta}^- \end{pmatrix} \). The linear model of UC\(^-\) is

\[
\dot{x}_2 = A_2 x_2 + B_2 y_3
\]
\[y_2 = C_2 x_2 \tag{4.88}\]

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where

\[
A_2 = \begin{pmatrix}
0 & \omega \\
-\omega & 0
\end{pmatrix}, \quad
B_2 = \frac{3}{2}k \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}, \quad
C_2 = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

The state vector, input, and output of the sequence extractor block are defined as

\[
x_3 = \begin{pmatrix}
i_\alpha^+ \\
i_\beta^+ \\
i_\alpha^- \\
i_\beta^-
\end{pmatrix}, \quad
u_3 = y_4 = \begin{pmatrix}
i_\alpha \\
i_\beta
\end{pmatrix}, \quad
\text{and } y_3 = \begin{pmatrix}
i_\alpha^- \\
i_\beta^-
\end{pmatrix}.
\]

With these definitions, the linear model of the sequence extractor is

\[
\dot{x}_3 = A_3 x_3 + B_3 y_4
\]

\[
y_3 = C_3 x_3
\]

(4.89)

where

\[
A_3 = \begin{pmatrix}
-\mu & -\omega & -\mu & 0 \\
\omega & -\mu & 0 & -\mu \\
-\mu & 0 & -\mu & \omega \\
0 & -\mu & -\omega & -\mu
\end{pmatrix}, \quad
B_3 = \mu \begin{pmatrix}
1 & 0 \\
0 & 1 \\
1 & 0 \\
0 & 1
\end{pmatrix}, \quad
C_3 = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
As for the output filter, define the state vector $x_4 = \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}$, input $u_4 = \begin{pmatrix} v_\alpha \\ v_\beta \end{pmatrix}$,

$\begin{pmatrix} v_\alpha \\ v_\beta \end{pmatrix}$, and output $y_4 = \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}$. Then, the LTI representation of the output filter is

$$x_4' = A_4 x_4 + B_4 (y_1 + y_2) - B_4 \begin{pmatrix} v_\alpha \\ v_\beta \end{pmatrix}$$

$$y_4 = C_4 x_4$$

(4.90)

where

$$A_4 = \frac{R}{L} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, B_4 = \frac{1}{L} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, C_4 = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$ 

The complete state-space model of the system for stability analysis is obtained by augmenting the above LTI models of the subsystems and is expressed as

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \begin{pmatrix} A_1 & O_{2\times4} & O_{2\times4} & B_1 C_4 \\ O_2 & A_2 & B_2 C_3 & O_2 \\ O_{4\times2} & O_{4\times2} & A_3 & B_3 C_4 \\ B_4 C_1 & B_4 C_2 & O_{2\times4} & A_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} O_2 \\ O_{4\times2} \\ -B_4 \end{pmatrix} \begin{pmatrix} v_\alpha \\ v_\beta \end{pmatrix}$$

(4.91)

where

$$O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, O_{2\times4} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}, O_{4\times2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. $$
The complete system may be represented as

\[
\dot{x} = Ax + Bu \tag{4.92}
\]

where the state vector is defined as \(x = [v^+_\alpha \ v^+_\beta \ v^-_\alpha \ v^-_\beta \ i^+_\alpha \ i^+_\beta \ i^-_\alpha \ i^-_\beta \ i_\alpha \ i_\beta]^T\),

\[
\text{input } u = \begin{pmatrix} v_{g\alpha} \\ v_{g\beta} \end{pmatrix},
\]

\[
A = \begin{pmatrix}
A_1 & O_2 & O_{2\times4} & B_1C_4 \\
O_2 & A_2 & B_2C_3 & O_2 \\
O_{4\times2} & O_{4\times2} & A_3 & B_3C_4 \\
B_4C_1 & B_4C_2 & O_{2\times4} & A_4
\end{pmatrix}, \quad B = \begin{pmatrix}
O_2 \\
O_2 \\
O_{4\times2} \\
-B_4
\end{pmatrix}
\]

The full system model has 10 state variables.

### 4.2.4 Complete Stability Analysis of Proposed Controller

Some design guideline used in the stability analysis of individual positive and negative sequence control units is used for the complete stability analysis. Specifically, and in order to derive a sample of results, we select \(R = 2\alpha L\) (for both \(UC^+\) and \(UC^-\) units), \(L = 5\, \text{mH}\), \(\mu = 300\) (for the sequence extractor), and \(k^+ = k^- = k\).

Figure 4.14 shows the eigenvalues of the system when \(P_{\text{ref}} = 0\), \(Q_{\text{ref}} = 0\), \(\alpha = 50\) and \(k\) is changed from 0 to \(\frac{RZ}{3L}\). When \(k = 0\), the four eigenvalues related to sequence extractor are at \(-\mu \pm \sqrt{k^2 - \omega_n^2}\), two eigenvalues at \(-\frac{R}{L}\), and four other eigenvalues at \(\pm j\omega_n\).
k is increased from 0 to $\frac{RZ}{3L}$, the eigenvalues move as shown in Figure 4.14. The figure confirms the stability of the closed-loop control system. It also shows that there are four dominant eigenvalues which have an approximate real values close to $-\alpha = -50$. Notice that the individual positive and negative sequence control units have eigenvalues exactly at $-\alpha$. But Fig. 4.14 shows the accurate picture which includes the coupling between these two control units as well as the coupling between them and the sequence extractor.

The above analysis is for $P_{\text{ref}} = 0$ and $Q_{\text{ref}} = 0$. The impact of the nonzero powers on the system’s eigenvalues is shown in Figure 4.15 where the parameters are set to $\alpha = 50$, $\mu = 300$, $R = 2\alpha L$, and $k = \frac{RZ}{3L}$. Figure 4.15 shows the robustness of the system to the change in power input. The level of change in eigenvalues, caused by the power change, is almost invisible in most cases.
Figure 4.15

Eigenvalues of the proposed system for nonzero powers
When a DER rides through a grid fault, it is required to adjust its real and reactive powers according to its corresponding regulating standards. Therefore, the DER control system must be able to determine the time interval of the fault as precisely and accurately as possible. This requires a fast and accurate fault detection algorithm which is the subject of this chapter.

The fault detection in power systems is a subject with long history and there are a number of methods and algorithms to do this task. In this research, we adopt the fault detection method proposed in [21]. This method is based on using a combination of EPLL and Wavelet techniques to detect the instant of fault in a fast way. Moreover, it is able to distinguish between the actual faults and the non-fault transients such as those arising after a capacitor bank switching in the power system. The method of [21] has proved to offer a robust performance.

5.1 Voltage Sag Types in Electric Power System

Electric power system is subjected to different types of faults. The common type of faults in the power system are the symmetrical faults such as three phase fault and three phase to ground fault and the unsymmetrical faults such as Single Line-to-Ground (SLG),
Line-to-Line (LL), and Line-Line-to-Ground (LLG) faults. These faults can occur in any part of the power system network. The interest is on the effect of these faults on the Point of Common Coupling (PCC) where the DERs are connected to the grid. These faults generally cause a voltage sag at PCC. A voltage sag is a decrease in the voltage magnitude normally accompanied with a change of phase angle of one or more phase voltages at PCC. The voltage sags are classified into different types as described in [10]. The type of voltage sag appearing at the PCC depends on the type of fault and the transformer winding type present in the power system network. Different types of voltage sags are shown in Figure 5.1. The dashed line shows the original three phase voltage phasor and the solid line shows the voltage phasor due to voltage sag. Here, the phase voltages are in per unit and V is the sag magnitude in per unit.

Type-A voltage sag is caused by three phase faults or three phase to ground faults. Type-B voltage sag is generally caused by SLG faults. Type-C voltage sag is caused by LL faults while Type-E voltage sag is caused by LLG faults. When there is a fault in any part of power system network, the voltage sag propagates throughout the network. The type of voltage sag may change during propagation because of the transformer windings or loads present in the network. Voltage sag appearing at the PCC may be of any type as shown in Figure 5.1. So the fault in the system can be simulated by one of the voltage sags of Figure 5.1.

Fault detection algorithm first detects the disturbance in the voltage at PCC usually characterized by the voltage sag at PCC. Besides grid faults, the voltage disturbances are also caused by other events such as capacitor switching, transformer energizing, and con-
Figure 5.1
Phasor diagram of (a) Type-A (b) Type-B (c) Type-C (d) Type-D (e) Type-E (f) Type-F and (g) Type-G voltage sags

nection of large induction motors in the power system network. In the fault detection method of [21], the voltage disturbance caused by any event in the power system is detected by an enhanced phase-locked loop (EPLL) system. We will used the same three phase enhanced phase-locked loop (EPLL-II) structure, that is used as part of the proposed
controller for the estimation of the positive and negative sequence components of the grid voltage, to perform this task. The error signal is generated by the EPLL-II structure with no delay and it is available instantly for further processing using wavelet transform.

The fault detection algorithm uses this error signal to detect the fault by wavelet based analysis as proposed on [21]. This algorithm can precisely distinguish the voltage disturbance caused by faults and by other events such as capacitor switchings. It is shown in [21] that the fault can be detected within $3.12\, ms$ of the occurrence of the fault in the worst case scenario.

### 5.2 Disturbance Detection

Any event causing voltage disturbance is instantly detected by the EPLL-II structure. The details of the EPLL-II structure is discussed in Appendix A. The error signal of EPLL-II instantly reflects any disturbance in the PCC voltage and can be used for quick detection of the events. Figure 5.2 shows, by way of an example, the grid voltage and the error signal from EPLL-II unit. The voltage disturbance due to fault occurs at $t = 0.4\, s$ and the event is reflected instantly on the error signal. Notice that the EPLL-II is implemented in $\alpha\beta$ domains and the errors show them in this domain.

### 5.3 Fault Detection using Wavelet Transform

After the disturbance detection, the error signal is further analyzed to distinguish between the fault and the capacitor switching events. The analysis includes wavelet transform of the error signal, feature extraction, and the decision making process. The wavelet trans-
Figure 5.2

Grid voltages and the error signals from EPLL-II unit

![Figure 5.2](image)

Figure 5.3

Wavelet transform realization using filter banks

![Figure 5.3](image)

Form is achieved using three stages of filtering as shown in Figure 5.3 where the filters are the Finite Impulse Response (FIR) filters realised as

\[ g[n] = \frac{1}{4}\delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{4}\delta[n-2] \]
\[ h[n] = -\frac{1}{4}\delta[n] + \frac{1}{2}\delta[n-1] - \frac{1}{4}\delta[n-2] \]

and \( \delta[n] \) is the discrete-time impulse function.

Based on the signals from wavelet transform block, features to identify the specific signatures of the event are extracted. The features are defined as

\[ F^x_i = \frac{s^x_i}{e^x}, \quad G^x_i = \frac{c^x_i}{e^x} \]

where, \( x = a, b, c \) is the phase and \( i = 1, 2, 3 \) denotes the stage of wavelet transform block where features are calculated. Reference [21] provides the mean and variance of the features for different types of faults and capacitor switching transients in a typical power system situation. The results are summarized in Table 5.1. The Gaussian probability density function (PDF) is calculated for the six features \((F^x_1, F^x_2, F^x_3, G^x_1, G^x_2, G^x_3)\) using

\[
p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-m)^2}{2\sigma^2}}
\]

and the mean and variance data from Table 5.1. Figure 5.4 shows the Gaussian PDF for the six features for faults and transients.

Maximum Likelihood criterion is used for the decision making process to distinguish between faults and the capacitor switching events. After the disturbance detection, the features \( f_1^x \) and \( g_1^x \) are calculated at the third sampling period, the features \( f_2^x \) and \( g_2^x \) at seventh sampling period, and features \( f_3^x \) and \( g_3^x \) at fifteenth sampling period. Let \( E \) be an event (fault or transients in this case) for which the features \( f_i^x \) and \( g_i^x \) are generated. Then the likelihood function for the event \( E \), considering equal probability for all events, is defined as

\[
p(F_i^x = f_i^x \& G_i^x = g_i^x | E) \quad (5.2)
\]
Table 5.1

Mean and Variance of the features

<table>
<thead>
<tr>
<th></th>
<th>Faults</th>
<th>Transients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^x$</td>
<td>0.000708</td>
<td>0.000015</td>
</tr>
<tr>
<td></td>
<td>0.018879</td>
<td>0.000045</td>
</tr>
<tr>
<td>$G_1^x$</td>
<td>0.994250</td>
<td>0.000310</td>
</tr>
<tr>
<td></td>
<td>0.940560</td>
<td>0.000027</td>
</tr>
<tr>
<td>$F_2^x$</td>
<td>0.008702</td>
<td>0.000135</td>
</tr>
<tr>
<td></td>
<td>0.206980</td>
<td>0.006039</td>
</tr>
<tr>
<td>$G_2^x$</td>
<td>0.987440</td>
<td>0.000273</td>
</tr>
<tr>
<td></td>
<td>0.694540</td>
<td>0.000688</td>
</tr>
<tr>
<td>$F_3^x$</td>
<td>0.011155</td>
<td>0.000355</td>
</tr>
<tr>
<td></td>
<td>0.019764</td>
<td>0.073045</td>
</tr>
<tr>
<td>$G_3^x$</td>
<td>0.897900</td>
<td>0.065004</td>
</tr>
<tr>
<td></td>
<td>0.105746</td>
<td>0.027727</td>
</tr>
</tbody>
</table>

where \( p \) is the conditional PDF of event \( E \) with respect to \( F_i^x \) and \( G_i^x \). Since the PDFs of all the feature are available as shown in Figure 5.4, the maximum value of the function of Equation (5.2) for the events (faults and transients) can be calculated. The decision of the event being fault or the transient due to capacitor switching is made using this maximum value. Fault detection times obtained in [21] are summarized in Table 5.2.

Table 5.2

Fault detection time

<table>
<thead>
<tr>
<th>Fault</th>
<th>( n = 3 ) 0.24ms</th>
<th>( 3 &lt; n \leq 7 ) 0.72ms</th>
<th>( 7 &lt; n \leq 15 ) 1.68ms</th>
<th>( 15 &lt; n \leq 27 ) 3.12ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLG</td>
<td>30%</td>
<td>50%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>LLG</td>
<td>76%</td>
<td>18%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>LL</td>
<td>71%</td>
<td>12.4%</td>
<td>15%</td>
<td>2.6%</td>
</tr>
</tbody>
</table>
Figure 5.4

Gaussian PDFs for the features

Figure 5.5

Fault detection time (example)
Figure 5.5 shows the simulation result of the fault detection algorithm in a simulated example. Here, Type-E voltage sag with 20% decrease in voltage magnitude is applied at $t = 0.4\ s$ and the event is detected within 0.2 $ms$ as denoted by event detection flag. The fault is detected in 0.4 $ms$ as shown by the fault detection flag.
CHAPTER 6
RESULTS

This chapter shows detailed results of the performance of the proposed control system. The results include digital simulations performed in PSIM software as well as real-time digital simulations in RTDS platform and experimental results from a laboratory experimental set-up.

The VSC block uses power electronic switches which impose very small simulation time step in order to include their accurate performance. This significantly lowers the simulation flexibility and power to study various scenarios. On the other hand, such switching phenomena have no impact on the performances of the proposed controllers. Therefore, we modeled the VSC as a controllable voltage source in the simulation studies. However, to ensure the validity of the results, some individual cases were performed using the actual model of VSC including its switches.

6.1 Simulation Results

The simulations are performed in PSIM [41]. A 3-phase 3-wire 2 KVA inverter with a dc bus voltage of 500 V is used for the simulations. The inverter is simulated using controlled voltage sources as well as the power electronic switches. The switching frequency of 10 kHz is considered and consequently the output filter of $L = 5 \text{ mH}$ is used. The grid is
a three-phase 208 V (line-to-line), 60 Hz. The grid is simulated by three phase ideal voltage source. The simulations are performed for different grid fault conditions such as three phase fault, SLG fault, LL fault, and LLG fault. The faults are simulated by the voltage sag at PCC. The three phase fault is simulated with Type-A voltage sag, SLG fault with Type-B voltage sag, LL fault with Type-C voltage sag, and LLG fault with Type-E voltage sag. The controller is implemented in digital with the sampling frequency of 10 kHz. The control parameters used are $\alpha = 50$, $R = 2\alpha L = 0.5 \, \Omega$, $k^+ = k^- = k = (RZ)/(3L) = 65$, $\mu = \alpha = 50$ for voltage sequence extractor unit, $\mu = 300$ for current sequence extractor unit, and $k_\omega = k^2/(16V^2) = 0.009$.

### 6.1.1 Performance without FRT Capability

Figure 6.1 shows the performance of a voltage controlled inverter without the fault ride-through capability. The universal controller of [26] is used for this and subsequent simulations in this chapter except stated otherwise. The Type-B voltage sag with a magnitude of 50% at $t = 0.4 \, s$ is applied. As it is observed from Figure 6.1, the voltage sag causes the flow of an excessively large and unbalanced current into the system. Moreover, the transferred power has large double frequency oscillations.

### 6.1.2 Performance of Proposed FRT Controller

The proposed controller is added to the voltage-controller inverter and its performance is presented here. The following scenario is defined and implemented for the simulations in this section.

- Initially, $P = 2 \, kW, Q = 0$. 


Figure 6.1

Performance of a typical voltage controller without fault ride through capability during a grid fault

- At $t = 0.4$ s, Type-A, Type-B, Type-C or Type-E voltage sag is applied (simulating different types of grid faults).
- At $t = 0.6$ s, $P$ is reduced to 1 kW.
- At $t = 0.75$ s, $Q$ is increased to 1 kVar.
- At $t = 0.9$ s, normal operating condition is restored.

Figure 6.2 shows the initial transients when there is a voltage sag of 50% at $t = 0.4$ s and the feed-forward signal for smooth transition is not added. The simulation confirms that the controller has the desired steady state operation but the transients are not desirable.

Figure 6.3 and Figure 6.4 show the performance of the proposed controller when Type-B voltage sag (simulating SLG fault) of 50% occurs and the feed-forward signal is enabled. It confirms that the proposed controller has a smooth transition from non-fault operating
Figure 6.2
Transients caused by a Type-B voltage sag of 50% (without feed-forward term for soft transition control)

Figure 6.3
Performance of the proposed controller during transition from normal to fault (Type-B sag) operating condition
condition to faulty operating condition. The proposed controller injects the balanced current during Type-B voltage sag and the subsequent real and reactive power commands are successfully executed by the controller. Figure 6.5 shows the transition from fault to normal operating condition for the same fault.

Figure 6.6 shows the smooth transition from normal to fault operating condition when a severe Type-B voltage sag with sag magnitude of 90% is applied. The performance of the controller is still desirable for this severe fault.

Figure 6.7 shows the performance of the proposed controller when Type-B voltage sag of 50% occurs and the VSC is modelled with the exact model of power electronic switches. The only difference with the responses from Figure 6.4, where VSC is modeled with ideal voltage sources, is the presence of high frequency switching ripples in Figure 6.7. This confirms that the controlled voltage-source modeling of the VSC conveys its control properties. Rest of the PSIM simulations are performed with VSC modeled with the ideal voltage sources.

Figure 6.8 and Figure 6.9 shows the performance of the proposed controller for a Type-E voltage sag (simulating LLG fault) of 50%. The simulation results confirm that the proposed controller injects a balanced current during Type-E voltage sag and it responds accurately and quickly to power commands. Figure 6.10 shows the transition when the fault is cleared. The transitions when fault occurs and when the fault is cleared are smooth as confirmed by the results. Figure 6.11 shows the smooth transition from normal to fault operating condition when a severe Type-E voltage sag with magnitude of 90% is applied.
Figure 6.4

Performance of the proposed controller in executing the real and reactive power commands during a Type-B voltage sag

Figure 6.5

Transition from fault (Type-B sag) to normal operating condition
Figure 6.6

Soft fault transition from normal to fault (Type-B sag of 90%)

Figure 6.7

Performance of the proposed controller for Type-B voltage sag (VSC modeled with power electronic switches)
Figure 6.8

Performance of the proposed controller during transition from normal to fault (Type-E sag) operating condition

Figure 6.9

Performance of the proposed controller in executing the real and reactive power commands during a Type-E voltage sag
Figure 6.10

Transition when fault (Type-E sag) is cleared

Figure 6.11

Soft fault transition from normal to fault (Type-E sag of 90%)
Figure 6.12 and Figure 6.13 shows the performance of the proposed controller for a Type-A voltage sag (simulating three phase fault) of 25%. The simulation results confirm that the proposed controller injects a balanced current during Type-A voltage sag and it responds accurately and quickly to power commands. Figure 6.14 shows the transition when the fault is cleared. The transitions when fault occurs and when the fault is cleared are smooth as confirmed by the results. Since Type-A voltage sag simulates the three phase fault (symmetrical fault), there are no oscillations in real and reactive powers. The performance results show the desirable operation of the proposed controller for the Type-A voltage sag.

Figure 6.15 and Figure 6.16 shows the performance of the proposed controller for a Type-C voltage sag of 50% where the phase angles in phase 'b' and phase 'c' are changed by 15°. The Type-C voltage sag is used to simulate the LL fault in the power system network. The simulation results confirm that the proposed controller injects a balanced current during Type-C voltage sag and it responds accurately and quickly to power commands. Figure 6.17 shows the transition when the fault is cleared. The transitions when fault occurs and when the fault is cleared are smooth as confirmed by the results.

The performance of the proposed controller is evaluated for the common types of grid faults. In all the fault scenarios presented above, as the figures clearly demonstrate, the current is balanced and the average real and reactive power commands are executed by the controller. Also the transitions during fault and fault clearance are smooth without excessive transients.
Figure 6.12

Performance of the proposed controller during transition from normal to fault (Type-A sag) operating condition

Figure 6.13

Performance of the proposed controller in executing the real and reactive power commands during a Type-A voltage sag
Figure 6.14

Transition when fault (Type-A sag) is cleared

Figure 6.15

Performance of the proposed controller during transition from normal to fault (Type-C sag) operating condition
Figure 6.16

Performance of the proposed controller in executing the real and reactive power commands during a Type-C voltage sag

Figure 6.17

Transition when fault (Type-C sag) is cleared
6.1.3 Implementation of Specific Grid FRT Standards

The proposed controller enables a DER to ride through the grid faults. The specific functions that the DER should satisfy during the fault are different in different grid codes associated with different countries as it was explained in Chapter 1. As a general rule, they need to feed (or absorb) reactive power depending on the level of voltage sag (or swell) and they also need to adjust the amount of their real power accordingly. The proposed controller can be adjusted to address such grid code requirements. The fault detection method, discussed in Chapter 5, quickly detects the instant of the occurrence of a fault. Then, the positive-sequence voltage magnitude which is estimated as a supplementary signal in the proposed controller (in the sequence extractor block) is used to determine the desired amount of real and reactive powers. These desired amounts are forwarded as reference values to the proposed controller.

This chapter presents the results of implementation of the grid codes of Germany and Denmark using the proposed controller. The grid code requirements for these two countries are discussed in Chapter 1.

6.1.3.1 Implementation of German Grid FRT Requirements

Two following scenarios are defined and considered to illustrate the performance of the proposed controller in addressing the German grid code for fault ride through situations.

1. Scenario I

   • Initially, \( P = 2 \text{ kW}, Q = 0 \).
• At $t = 0.4\,\text{s}$, Type B or Type E voltage sag is applied where voltage is reduced to $V_p = 0.5\,\text{pu}$.

• At $t = 0.5\,\text{s}$ $V_p = 0.7\,\text{pu}$.

• At $t = 0.6\,\text{s}$ $V_p = 0.8\,\text{pu}$.

• At $t = 0.7\,\text{s}$ $V_p = 0.9\,\text{pu}$.

• At $t = 0.8\,\text{s}$ $V_p = 1.0\,\text{pu}$.

2. Scenario II

• Initially, $P = 2\,\text{kW}, Q = 0$.

• At $t = 0.4\,\text{s}$, Type B or Type E voltage sag is applied where voltage is reduced to $V_p = 0\,\text{pu}$.

• At $t = 0.5\,\text{s}$ $V_p = 0.6\,\text{pu}$.

• At $t = 0.6\,\text{s}$ $V_p = 0.8\,\text{pu}$.

• At $t = 0.7\,\text{s}$ $V_p = 0.9\,\text{pu}$.

• At $t = 0.8\,\text{s}$ $V_p = 1.0\,\text{pu}$.

Scenario II is basically a more severe version of Scenario I. Figures 6.18, 6.19, 6.20, and 6.21 show the performance of the proposed controller for German grid code. The controller injects the reactive current when the fault is detected. The active power is reduced while injecting the reactive power. The reactive power injection is limited to 40% of the rating according to the German code requirement for wind power plants. The active power injection is resumed to the pre-fault value as soon as the fault is cleared.
Figure 6.18

LVRT performance for German grid code with Type-B voltage sag (Scenario I)

Figure 6.19

LVRT performance for German grid code with Type-B voltage sag (Scenario II)
Figure 6.20

LVRT performance for German grid code with Type-E voltage sag (Scenario I)

Figure 6.21

LVRT performance for German grid code with Type-E voltage sag (Scenario II)
6.1.3.2 Implementation of Danish Grid FRT Requirements

Two following scenarios are defined and considered to illustrate the performance of the proposed controller in addressing the Danish grid code for fault ride through situations.

1. Scenario I
   - Initially, $P = 2$ kW, $Q = 0$.
   - At $t = 0.4$ s, Type B or Type E voltage sag is applied where voltage is reduced to $V_p = 0.5$ pu.
   - At $t = 0.5$ s $V_p = 0.7$ pu and at $t = 0.6$ s $V_p = 0.8$ pu.
   - At $t = 0.7$ s $V_p = 0.9$ pu and at $t = 0.8$ s $V_p = 1.0$ pu.

2. Scenario II
   - Initially, $P = 2$ kW, $Q = 0$.
   - At $t = 0.4$ s, Type B or Type E voltage sag is applied where voltage is reduced to $V_p = 0$ pu.
   - At $t = 0.5$ s $V_p = 0.5$ pu and at $t = 0.6$ s $V_p = 0.7$ pu.
   - At $t = 0.7$ s $V_p = 0.9$ pu and at $t = 0.8$ s $V_p = 1.0$ pu.

Scenario II is basically a more severe version of Scenario I. Figures 6.22, 6.23, 6.24, and 6.25 show the performance of the proposed controller for Danish grid code. The controller injects the positive sequence reactive current when the fault is detected. The active power is reduced while injecting the reactive power. Up to 100% reactive power injection is permitted for the voltage drop to 0.5 pu and less. The active power injection is resumed to the pre-fault value as soon as the fault is cleared.
Figure 6.22
LVRT performance for Danish grid code with Type-B voltage sag (Scenario I)

Figure 6.23
LVRT performance for Danish grid code with Type-B voltage sag (Scenario II)
Figure 6.24

LVRT performance for Danish grid code with Type-E voltage sag (Scenario I)

Figure 6.25

LVRT performance for Danish grid code with Type-E voltage sag (Scenario II)
6.2 Real-Time Results

Real-time results are obtained from the Real Time Digital Simulator (RTDS) [29]. The inverter circuit, grid model and the controller units are developed in the RSCAD software and implemented in the RTDS hardware. A 3-phase 3-wire 2 kVA inverter with DC bus voltage of 500 V is modeled in RSCAD using small time step model of the controlled voltage sources. The small time step model runs at a time step of $1 - 3 \, \mu s$ when implemented in RTDS hardware. The output filter of $L = 5 \, \text{mH}$ is used. The 3-phase 60 Hz, 120 V (RMS, line to neutral) grid is considered. The grid is modelled with the small time step model of a three-phase voltage source. The faults are simulated using different types of voltage sag at PCC. The controller is modeled in RSCAD and implemented in the RTDS at the time step of $50 \, \mu s$. The control parameters used are $\alpha = 50$, $R = 2\alpha L = 0.5 \, \Omega$, $k = (RZ)/(3L) = 65$, $\mu = \alpha = 50$ for voltage sequence extractor unit, $\mu = 300$ for current sequence extractor unit, and $k_\omega = k^2/(16V^2) = 0.009$. The real-time results are obtained for the same scenarios used for the PSIM simulations.

Figure 6.26 shows the real-time responses of the proposed controller when a Type-B voltage sag occurs. The figure confirms generation of balanced currents. Figures 6.27 and 6.28 show responses of the real-time system to real and reactive power commands. The commands are faithfully tracked. The pulsation levels and the transient response times of real-time and simulation results comply closely. Figure 6.29 show the real-time result of the transition when fault is cleared. Figure 6.30 shows the transients when a severe Type-B voltage sag of 90% is applied.
Figure 6.26
Real-time results of Type-B voltage sag at $t = 0.04$ s ($P = 2$ kW, $Q = 0$)

Figure 6.27
Real-time results of Type-B voltage sag (real power jump from 2 kW to 1 kW, $Q = 0$ kVar)
Figure 6.28

Real-time results of Type-B voltage sag (reactive power jump from zero to 1 kVar, \( P = 1 \text{kW} \))

Figure 6.29

Real-time results showing transients when fault (Type-B sag) is cleared
Real-time results showing transients when severe voltage sag (Type-B sag of 90%) is applied

The real-time performance of the proposed controller during LLG fault is presented with Type-E voltage sag simulating the LLG fault. Figure 6.31 shows the real-time responses of the proposed controller when a Type-E voltage sag of 50% occurs. The figure confirms generation of balanced currents even during the grid fault situation. Figures 6.32 and 6.33 show responses of the real-time system to track real and reactive power commands. Figure 6.34 shows the real-time result of the transition when fault is cleared. Figure 6.35 shows the transients when a severe Type-E voltage sag of 90% is applied.

The real-time performance of the proposed controller is evaluated for three phase faults with Type-A voltage sag at PCC. Figure 6.36 shows the real-time responses of the proposed controller when a Type-A voltage sag of 25% occurs. Figures 6.37 and 6.38 show
Figure 6.31

Real-time results of Type-E voltage sag at $t = 0.04$ s ($P = 2$ kW, $Q = 0$)

responses of the real-time system to real and reactive power commands. Figure 6.39 show the transition when fault is cleared.

Figure 6.40 shows the real-time responses of the proposed controller when a Type-C voltage sag of 50% occurs. Figures 6.41 and 6.42 show responses of the real-time system to real and reactive power commands. Figure 6.43 show the transition when fault is cleared. The results show that the proposed controller injects balanced positive sequence current and the real and reactive power commands are followed. The transition from non-fault to fault and from fault to non-fault operating conditions are smooth as demonstrated in the results.
Figure 6.32

Real-time results of Type-E voltage sag (real power jump from 2 kW to 1 kW, $Q = 0$ kVar)

Figure 6.33

Real-time results of Type-E voltage sag (reactive power jump from zero to 1 kVar, $P = 1$ kW)
Figure 6.34
Real-time results showing transients when fault (Type-E sag) is cleared

Figure 6.35
Real-time results showing transients when severe voltage sag (Type-E sag of 90%) is applied

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Figure 6.36

Real-time results of Type-A voltage sag at $t = 0.04$ s ($P = 2$ kW, $Q = 0$)

Figure 6.37

Real-time results of Type-A voltage sag (real power jump from 2 kW to 1 kW, $Q = 0$ kVar)
Figure 6.38

Real-time results of Type-A voltage sag (reactive power jump from zero to 1 kVar, $P = 1$ kW)

Figure 6.39

Real-time results showing transients when fault (Type-A sag) is cleared
Real-time results of Type-C voltage sag at $t = 0.04$ s ($P = 2$ kW, $Q = 0$)

Real-time results of Type-C voltage sag (real power jump from 2 kW to 1 kW, $Q = 0$ kVar)
Figure 6.42

Real-time results of Type-C voltage sag (reactive power jump from zero to 1 kVar, $P = 1 \text{ kW}$)

Figure 6.43

Real-time results showing transients when fault (Type-C sag) is cleared
6.3 Proposed Controller Added to other Voltage Control Methods

The performances of the proposed FRT controller when added to some other existing voltage control methods are presented using PSIM simulations in this section. The other voltage control methods used are Synchronverter and Virtual Synchronous Generator (VSG).

The simulation model of the Synchronverter based on [39] is built and the proposed auxiliary controller is added. Figure 6.44 shows the performance of the original Synchronverter during Type-B voltage sag of 50%. The result clearly shows that over-current happens and the generated current is highly unbalanced during the fault. Figure 6.45, 6.46, and 6.47 show the performance of the Synchronverter when the proposed auxiliary controller is added. Figure 6.45 shows the transition from normal operation to fault while Figure 6.47 shows the transition when the fault is cleared. The results show that the proposed controller successfully implements the LVRT control for Synchronverter.

The simulation model of the VSG based on [46] is built and its performance without and with the proposed auxiliary controller is studied and presented in this section. Figure 6.48 shows the performance of the original VSG during Type-B voltage sag of 50%. The results clearly show large and highly unbalanced currents during the fault. Figure 6.49, 6.50, and 6.51 show the performance of the modified VSG when the proposed auxiliary controller is added. Figure 6.49 shows the transition from normal operation to fault while Figure 6.51 shows the transition when the fault is cleared. The results show that the proposed controller successfully implements the LVRT control for VSG as well.
Figure 6.44

Performance of the Synchronverter without LVRT control

Figure 6.45

Performance of the modified Synchronverter during transition from normal operation to fault (Type-B sag)
Performance of the modified Synchronverter during a Type-B voltage sag (response to real and reactive power commands)

Performance of the modified Synchronverter in transition when fault (Type-B voltage sag) is cleared
Figure 6.48

Performance of the VSG without LVRT control

Figure 6.49

Performance of the modified VSG during transition from normal operation to fault (Type-B sag)
Figure 6.50

Performance of the modified VSG during a Type-B voltage sag

Figure 6.51

Performance of the modified VSG in transition when fault (Type-B voltage sag) is cleared
6.4 Experimental Results

The experimental setup is based on a 1.2 kVA three-phase inverter prototype with the DC bus voltage of 200 V. The controller is implemented in dSPACE MicroLabBox digital controller and programmed using Matlab/Simulink interface [16]. The dSPACE is interfaced with the inverter board through communication cable where the gating signals are forwarded to the gate drivers. The grid parameters are considered at 60 V (rms, line-to-ground) and 60 Hz. The grid is simulated using a three-phase programmable Chroma ac source which allows modeling of various grid faults [14]. The inverter is connected to the grid using an $LC$ output filter with $L = 5 \text{ mH}$ and $C = 2 \mu\text{F}$. The switching frequency of the inverter is 10 kHz and the sampling frequency of the digital controller is at 20 kHz. Figure 6.52 shows the snapshot of the components of the experimental setup.
Experimental results for Type-B and Type-A voltage sags of 25% are presented. In all these results, the inverter is initially operating at $P = 450$ W and $Q = 150$ Var before the occurrence of the fault. Ability of the inverter in riding through the fault, executing real and reactive power commands during the fault and transition out from the fault is illustrated.

Figure 6.53 shows the transition from normal to faulty operation for Type-B voltage sag of 25%. Figure 6.54 shows the transition from fault to normal operating condition. Figures 6.55 and 6.56 show the response of the controller to the real and reactive power commands during Type-B voltage sag.

Figure 6.57 shows the transition from normal to faulty operation for Type-A voltage sag of 25%. Figure 6.58 shows the transition from fault to normal operating condition. Figures 6.59 and 6.60 show the response of the controller to the real and reactive power commands during Type-A voltage sag.

The experimental results confirm that the proposed controller can ride through the faults and execute the power commands. The proposed controller employs a feed-forward control term from the grid voltage in order to reduce the transients subsequent to the happening of fault. This can adversely impact the inverter’s current quality if the measurements are not very accurate and/or impacted by sensor noise. In the presented experimental results, we had to slow down the controller in order to improve the current quality which was degraded due to sensor noise and inaccuracies. This causes slightly larger transient responses as compared with the simulation and real-time results.

The experimental results for Type-C voltage and Type-E voltage sag could not be obtained due to the limitations of the three phase programmable ac source used to simulate
Figure 6.53

Experimental results: fault ride-through transients for a 25 % sag of Type-B

Figure 6.54

Experimental results: fault clearance transients for a 25 % sag of Type-B
Figure 6.55
Experimental results: tracking real power jump from 750 W to 450 W during Type-B voltage sag

Figure 6.56
Experimental results: tracking reactive power jump from 150 Var to 450 Var during Type-B voltage sag
Figure 6.57

Experimental results: fault ride-through transients for a 25 % sag of Type-A

Figure 6.58

Experimental results: fault clearance transients for a 25 % sag of Type-A
Figure 6.59
Experimental results: tracking real power jump from 750 W to 450 W during Type-A voltage sag

Figure 6.60
Experimental results: tracking reactive power jump from 150 Var to 450 Var during Type-A voltage sag
grid in the experimental set-up. Therefore, the experimental results for LL fault and LLG faults are not presented.

6.5 Conclusions from Simulation, Real-time and Experimental Results

This chapter first presented the performance of a voltage controller (Universal Controller) without LVRT control strategy. Then the proposed controller is added to the voltage controller for LVRT control. The simulation model was developed in PSIM software. The performance results of the proposed controller for different fault conditions represented by different types of voltage sags were presented. The fault detection algorithm was then implemented to evaluate the performance of proposed controller to follow German and Danish grid codes.

Real-time performance results of the proposed controller was obtained from the RTDS. The simulation scenarios used in PSIM simulations were also used for real-time simulations. The real-time results showed the close compliance with the PSIM simulation results. The performance of the proposed controller with some other voltage control methods was also evaluated. The performance results of the proposed controller when added to Synchronverter and VSG for the LVRT control were presented specifically.

Experimental results were collected from a low power laboratory prototype of the proposed controller where dSPACE is used as the control implementation platform and interface. The grid was simulated using a three-phase Chroma programmable voltage source. Experimental results also confirm the ability of the proposed controller to ride through the faults and execute power commands during the faults.
CHAPTER 7

CONCLUSION AND FUTURE WORK

The grid fault ride-through capability for the inverters controlled with a voltage control algorithm is researched in this dissertation. For such capability, normally the current references are available. Meanwhile, voltage-controlled inverters are becoming dominant due to their more universal features enabling them to operate in microgrid applications and their ability to provide direct voltage support.

This work proposes a new control system that enables the voltage-controlled three-phase grid-connected inverters to ride through the grid faults. The proposed controller can be added to different voltage control algorithms. The voltage control method called the universal controller is used as the basis for the developments in this research.

The proposed control structure consists of a main component, that is inspired from the original developments of the universal component, and a current sequence extraction unit. A feed-forward term that uses a voltage sequence extraction unit is added to achieve soft transition at the fault instant. A fault detection algorithm is also adopted to determine the instant of the fault for the purpose of implementing specific grid standards. Subsequently, the German and Danish grid codes are implemented and the ability of the proposed controller in addressing them is illustrated. Complete stability analysis of the proposed con-
The proposed controller is performed mathematically and a new modeling methodology is also developed. This model which represents the nonlinear system with an LTI model proved to be significantly useful for analysis and design of the control system.

Extensive numerical studies are performed using computer simulation in PowerSim software as well as the real-time RTDS platform and the results are presented and discussed to show performance of the proposed controller in various operating conditions. The experimental results from the laboratory experimental set-up are also presented.

**Future Work**

1. *Extend the proposed controller to three-phase systems with neutral connection.*

   The proposed controller of this dissertation is developed for three-phase three-wire systems with no neutral connection which is the dominant type of inverters in practice. If the neutral connection is available in some practical applications, the inverter may be upgraded to control the zero-sequence current as well. This will add an additional degree of freedom to the control system. This degree of freedom can, for instance, be used to remove power pulsations in real or reactive components as it is done in [30] for current-controlled inverters. This topic has not been addressed in the literature for the voltage-controlled inverters.

2. *Extend the proposed LTI modeling to weak grid conditions.*

   The proposed LTI model of this paper assumes a stiff grid with fixed frequency. While this model is approximately valid for practical grids, the extent of its validity shrinks as the grid becomes smaller and weaker. A study of the extent of
the validity of this model and, if need be, development of a more general model can be the topic of another future work.

3. **Transient response improvement subsequent to a fault.**

   The fault ride-through capability can significantly help in maintaining the grid stability when the transient faults occur. The fast and smooth response of the inverter subsequent to the fault is critical. The proposed controller of this dissertation achieves a highly smooth and relatively fast response. The smoothness is achieved thanks to the feed-forward control. However, the transient responses take about two cycles to settle. This delay is partly due to the voltage sequence extractor and partly due to the main controller processing the negative sequence voltage. The grid voltage magnitude estimated in the voltage sequence extractor is used to adjust the reactive power reference and this seems to be the dominant cause for the mentioned delay. A research may be conducted to find out better (or faster) ways of estimating the grid voltage magnitude to reduce the delay.

4. **Current quality improvement.**

   In practical conditions where the grid voltage is distorted and the measurement sensors add errors and noise to the grid voltage measurements, the feed-forward term used to improve the transient responses may cause degradation of current quality. This can be the topic for a future work related to the proposed controller.

5. **Weak DC link studies.**

   In the developments of the proposed controller in this dissertation, a stiff DC side is considered for the inverter. In renewable energy applications, this as-
sumption may or may not be acceptable depending on the size of energy storage used. A detailed study appears to be necessary to determine the short-term storage requirements and its relationship with the fault conditions. When the fault happens and the power pulsations are produced, they reflect on the DC link as its pulsations. If not confined, such oscillations can push the DC variable beyond acceptable limits.
REFERENCES


APPENDIX A

THREE PHASE ENHANCED PHASE-LOCKED LOOP
A.1 Structure of Three-Phase Enhanced Phase-Locked Loop

Structure of the Three-Phase Enhanced Phase-Locked Loop (EPLL-II) designed in $\alpha\beta$ reference frame is shown in Figure A.1. EPLL-II structure is used for the estimation of the positive and negative sequence components of the voltage and the current [22]. The EPLL-II structure can work with the unbalanced input signal and double-frequency oscillation due to negative sequence component is eliminated. This feature of EPLL-II structure has a great advantage in the power system applications where unbalanced voltages are frequently caused by asymmetrical faults.

Figure A.1

EPLL-II structure in stationary frame
A.2 Mathematical Derivation of EPLL-II Structure

Structure of EPLL-II is shown in Figure A.1. Consider an input signal $u_{\alpha\beta}$ in $\alpha\beta$ reference frame. Define the signals $y_{\alpha\beta}$ as the estimate of positive sequence component of $u_{\alpha\beta}$ and $z_{\alpha\beta}$ as the estimate of negative sequence component of $u_{\alpha\beta}$ where,

\[
\begin{bmatrix}
y_{\alpha} \\
y_{\beta}
\end{bmatrix} = \begin{bmatrix} U_p \cos \psi_p \\
U_p \sin \psi_p \end{bmatrix} = U_p C_2(\psi_p) \tag{A.1}
\]

and

\[
\begin{bmatrix}
z_{\alpha} \\
z_{\beta}
\end{bmatrix} = \begin{bmatrix} U_n \cos \psi_n \\
-U_n \sin \psi_n \end{bmatrix} = U_n C_2(-\psi_n) \tag{A.2}
\]

where, $U_p$ and $U_n$ are the estimate of the magnitude of positive sequence signal and negative sequence signal and $\psi_p$ and $\psi_n$ are the estimate of their phase angles. The vectors $C_2(\cdot)$ and $S_2(\cdot)$ of EPLL-II structure are defined as

\[
C_2(\theta) = \begin{bmatrix} \cos \theta \\
\sin \theta \end{bmatrix}, \quad S_2(\theta) = \begin{bmatrix} -\sin \theta \\
\cos \theta \end{bmatrix}. \tag{A.3}
\]

The error signal of EPLL-II structure, $e_{\alpha\beta}$ is defined as

\[
e_{\alpha\beta} = u_{\alpha\beta} - y_{\alpha\beta} - z_{\alpha\beta}. \tag{A.4}
\]

The EPLL-II structure is derived using an optimization approach. The cost function for the optimization is defined as

\[
J = \frac{1}{2} e_{\alpha\beta}^T e_{\alpha\beta}
\]

\[
= \frac{1}{2} [u_{\alpha\beta} - U_p C_2(\psi_p) - U_n C_2(-\psi_n)]^T [u_{\alpha\beta} - U_p C_2(\psi_p) - U_n C_2(-\psi_n)] \tag{A.5}
\]
The gradient descent method $\dot{x} = -\mu \frac{\partial J(x)}{\partial x}$ is used to derive the differential equation of the EPLL-II structure. The equations of EPLL-II structure are expressed as

\begin{align*}
\dot{U}_p &= \mu_1 C_2(\psi_p)^T e_{\alpha\beta} \quad \text{(A.6)} \\
\dot{\omega} &= \frac{\mu_2}{U_p} S_2(\psi_p)^T e_{\alpha\beta} \quad \text{(A.7)} \\
\dot{\psi}_p &= \omega + \frac{\mu_3}{U_p} S_2(\psi_p)^T e_{\alpha\beta} \quad \text{(A.8)} \\
\dot{U}_n &= \mu_4 C_2(-\psi_n)^T e_{\alpha\beta} \quad \text{(A.9)} \\
\dot{\psi}_n &= \omega - \frac{\mu_5}{U_n} S_2(-\psi_n)^T e_{\alpha\beta} \quad \text{(A.10)}
\end{align*}

### A.3 Linear Model of EPLL-II Structure

The EPLL-II system becomes linear time invariant system (LTI system) when the frequency estimation loop is disabled, $\mu_2 = 0$ and $\mu_1 = \mu_3 = \mu_4 = \mu_5 = \mu$.

Consider the differential equation for $y_\alpha$

\begin{align*}
y_\alpha &= \dot{U}_p \cos \psi_p - \dot{\psi}_p U_p \sin \psi_p \\
&= \mu C_2(\psi_p)^T e_{\alpha\beta} \cos \psi_p - \omega U_p \sin \psi_p - \mu S_2(\psi_p)^T e_{\alpha\beta} \sin \psi_p \\
&= -\omega y_\beta + \mu e_\alpha \quad \text{(A.11)}
\end{align*}

Similarly,

\begin{align*}
y_\beta &= \dot{U}_p \sin \psi_p + \dot{\psi}_p U_p \cos \psi_p \\
&= \mu C_2(\psi_p)^T e_{\alpha\beta} \sin \psi_p + \omega U_p \cos \psi_p + \mu S_2(\psi_p)^T e_{\alpha\beta} \cos \psi_p \\
&= \omega y_\alpha + \mu e_\beta \quad \text{(A.12)}
\end{align*}
\[ z_\alpha = \dot{U}_n \cos \psi_n - \dot{\psi}_n U_n \sin \psi_n \]
\[ = \mu C_2 (-\dot{\psi}_n)^T e_{\alpha \beta} \cos \psi_n - \omega U_n \sin \psi_n + \mu S_2 (-\dot{\psi}_n)^T e_{\alpha \beta} \sin \psi_n \]
\[ = \omega z_\beta + \mu e_\alpha \quad \text{(A.13)} \]

\[ \dot{z}_\beta = -\dot{U}_n \sin \psi_n - \dot{\psi}_n U_n \cos \psi_n \]
\[ = -\mu C_2 (-\dot{\psi}_n)^T e_{\alpha \beta} \sin \psi_n - \omega U_n \cos \psi_n + \mu S_2 (-\dot{\psi}_n)^T e_{\alpha \beta} \cos \psi_n \]
\[ = -\omega z_\alpha + \mu e_\beta \quad \text{(A.14)} \]

Equations (A.11), (A.12), (A.13), and (A.14) are expressed as
\[
\begin{pmatrix}
    y_\alpha \\
    y_\beta \\
    z_\alpha \\
    z_\beta
\end{pmatrix}
= \begin{pmatrix}
    \begin{pmatrix}
        0 & -\omega & 0 & 0 \\
        \omega & 0 & 0 & 0 \\
        0 & 0 & 0 & \omega \\
        0 & 0 & -\omega & 0
    \end{pmatrix}
    &
    \begin{pmatrix}
        y_\alpha \\
        y_\beta \\
        z_\alpha \\
        z_\beta
    \end{pmatrix}
    &
    \begin{pmatrix}
        \mu & 0 \\
        0 & \mu \\
        \mu & 0 \\
        0 & \mu
    \end{pmatrix}
    &
    \begin{pmatrix}
        e_\alpha \\
        e_\beta
    \end{pmatrix}
\end{pmatrix} +
\begin{pmatrix}
    \begin{pmatrix}
        u_{\alpha \beta} \\
        e_{\alpha \beta}
    \end{pmatrix}
    &
    \begin{pmatrix}
        \mu I_2 \\
        \frac{1}{s} I_2
    \end{pmatrix}
    &
    \begin{pmatrix}
        \Omega_n \\
        -\Omega_n
    \end{pmatrix}
\end{pmatrix}
\]
\[ \text{(A.15)} \]

Figure A.2

LTI model of EPLL-II structure in stationary frame
\[
\begin{pmatrix}
\dot{y}_\alpha \\
\dot{y}_\beta \\
\dot{z}_\alpha \\
\dot{z}_\beta
\end{pmatrix} =
\begin{pmatrix}
-\mu & -\omega & -\mu & 0 \\
\omega & -\mu & 0 & -\mu \\
-\mu & 0 & -\mu & \omega \\
0 & -\mu & -\omega & -\mu
\end{pmatrix}
\begin{pmatrix}
y_\alpha \\
y_\beta \\
z_\alpha \\
z_\beta
\end{pmatrix} +
\begin{pmatrix}
\mu & 0 \\
0 & \mu \\
\mu & 0 \\
0 & \mu
\end{pmatrix}
\begin{pmatrix}
u_\alpha \\
u_\beta
\end{pmatrix}
\tag{A.16}
\]

Equation (A.15) represents the LTI model of EPLL-II structure. Figure A.2 shows the block diagram of LTI model of EPLL-II where,

\[
\Omega_n =
\begin{pmatrix}
0 & -\omega \\
\omega & 0
\end{pmatrix}
\]